Uncertainty, Misallocation and the Life-cycle Growth of Firms*

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September 5, 2024

Abstract

We propose a decomposition of static misallocation that distinguishes between idiosyncratic uncertainty and ex ante misallocation generated by tax-like distortions. Using profits-to-wage-bill ratios and value-added-to-wage-bill ratios, we can identify the two sources of misallocation. In the comprehensive Finnish firm-level data, uncertainty accounts for 41% of aggregate misallocation and has a strong decreasing age-dependent trend in it. We show that our results are quantitatively consistent with a life-cycle model of firm growth that incorporates learning. According to the dynamic model, uncertainty suppresses output by 8-12%, while ex ante misallocation has a 40% negative effect on output.

Keywords: firm dynamics, uncertainty, misallocation

JEL Codes: D24, E23, L11, O47

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* We are indebted to William Kerr, Vincent Sterk, Akos Valentinyi and our discussant Paul Klein for detailed feedback. We would also like to thank Philippe Aghion, Niku Määttänen, Morten Ravn, Matti Sarvimäki, Petr Sedláček, Juhana Siljander, Juuso Välimäki and conference participants at CEF, EEA-ESEM, HMRAD, Lisbon Macro Workshop and Nordic Summer Symposium in Macroeconomics for helpful comments and suggestions. Oskari Vähämaa acknowledges financial support from the Emil Aaltonen Foundation. First version: May 2021.

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1 Introduction

In order to understand the determination of aggregate output, it is paramount to have a clear picture of factors determining the aggregate total factor productivity.[1] A seminal paper of Restuccia and Rogerson (2008) illustrates that inefficient allocation of input factors across production units - misallocation - can have severe effects on TFP. To evaluate the empirical relevance of this channel, a popular indirect approach, pioneered by Hsieh and Klenow (2009) (henceforth HK), has been to measure marginal products of labor and capital using firm-level micro data.[2] If an economy's dispersion of marginal products, at least for the part that exceeds the US benchmark, reflects misallocation, the reallocation of input factors could lead to a significant increase in TFP and output. However, there are other factors that can generate dispersion in marginal products that are not necessarily directly related to inefficient allocation of input factors across producers.[3]

In this paper, we propose a decomposition of static misallocation that distinguishes between uncertainty and ex ante misallocation generated by tax-like distortions. Our approach builds upon the static framework of HK, the key difference being that in our setup, firms may not have perfect knowledge of their fundamentals (such as productivity or demand) when making input decisions. Additionally, similar to HK, firms face idiosyncratic revenue distortions. As a result, in our accounting framework, ex post resources might appear inefficiently allocated either because of the variation in firms' prediction errors (the uncertainty channel), because of the variation in the revenue distortions (the misallocation channel) or because of the covariance between the two.

[1] See, e.g., Klenow and Rodriguez-Clare (1997) for the importance of TFP in explaining differences in output across countries.

[2] See, e.g., Bayer et al. (2018), Busso et al. (2013) and Cirera et al. (2020) for recent examples of this approach.

[3] For example, measurement errors, different production/demand structures and adjustment costs costs could affect the measured misallocation. See e.g. Bils et al (2021), Rotemberg and White (2021), Gollin and Udry (2020), Haltiwanger et al. (2018), Bartelsman et al. (2013) and Asker et al. (2014).

Our indirect approach requires more information than the HK approach. This is because the differences in marginal products, measured with value-added-to-wage-bill ratios, could be generated by either tax-like distortions or prediction errors. To solve this identification problem, we utilize profits-to-wage-bill ratios, which allow us to pin down prediction errors. The intuition behind this is that tax-like distortions affect profits and employment in a similar way, implying that their ratio is independent of the revenue wedge. On the other hand, the effects of imperfect information are asymmetric: profits depend on realized fundamentals and expected fundamentals, while employment only depends on expected fundamentals. Finally, value-added-to-wage-bill ratios together with prediction errors give us the tax-like wedges.

We use our methodology to quantify the relative importance of idiosyncratic uncertainty in Finnish firm-level data. We show that in administrative data covering nearly all Finnish firms, 41% of the variation in the HK revenue distortion is accounted for by uncertainty. The variation in the tax-like distortion, our indirect measure of misallocation[4], explains 36% of the total variation.

We also observe that uncertainty strongly diminishes with firm age. The dispersion in the prediction error is more than halved when one moves from new businesses to firms that have been operating for a decade. This strong trend is mirrored in a similar one in the dispersion of the HK revenue wedge.[5] Misallocation, measured by the variation of the revenue wedge, on the other hand, is practically constant for all age brackets. Decomposing the HK dispersion conditional on age requires a large set of firms with varying ages, thus observing almost the whole population of firms is important.

Our accounting framework hints that uncertainty, especially in early life-cycle, might play an important role in determining the aggregate TFP. However, to be able to evaluate the relative importance of uncertainty and misallocation for aggregate outcomes, we need to move beyond static

[4]As in Restuccia and Rogerson (2008), we model idiosyncratic tax-like distortions as the source of misallocation.

[5]This is in line with Eslava et al (2023), who observe that idiosyncratic distortions are particularly important for young plants in Colombian establishment-level data.

calculations. To this end, we set up a life-cycle model of firm growth with entry and exit where the age-size distribution of firms is endogenously determined. Our model augments the learning structure of Jovanovic (1982) to a GE framework similar to Hopenhayn and Rogerson (1993) and Melitz (2003).

The key features of our model are age-dependent uncertainty, convex adjustment costs and tax-like wedges, all of which reduce the efficiency of resource allocation. In line with our static exercise, inputs are chosen before firms know their current period TFP.[6] Moreover, this productivity is a combination of a persistent component and a transitory one, which firms are unable to separate from each other. They use Bayesian learning to form a forecast about their current persistent productivity. This structure implies that the dispersion of marginal products is smaller for older firms whose forecasts are more precise. We also allow for adjustment costs, which further generate age-dependent dispersion, as some authors have suggested this channel of misallocation might be important.[7] In addition, firms' input decisions depend on revenue distortions that we use to model misallocation.

Even though our model is relatively parsimonious, it is flexible enough to explain the salient life-cycle features of the Finnish firm-level data that are not directly related to misallocation.[8] We utilize this feature in quantitatively disciplining our model, i.e, we match the model to the growth profiles of young and old firms, the size distribution of old firms and the basic selection patterns.

We use our calibrated model to redo our indirect estimates of static misallocation with an artificial data and demonstrate that our model is consistent with the observed patterns in the data. Firstly, our model accounts for 90% of the observed variation in the HK revenue distortion with the uncertainty and ex ante misallocation shares also in line with the Finnish data. Secondly, we find that the model generates a decreasing age-dependent

[6]With risk neutral firms, this is the equivalent of assuming one period time-to-build for inputs, a setup used recently, e.g., by Boar et al (2022).

[7]See, e.g., Eslava et al (2023) or Asker et al. (2014).

[8]Recently Arkolakis et al (2018) have explained the life-cycle growth of firms in equilibrium setups with Bayesian learning, while Clementi and Plazzo (2013) use adjustment costs for the same purpose.

trend in uncertainty that is quantitatively highly similar to the one observed in the data.

By using our calibrated model to conduct counterfactual analyses, we can evaluate the importance of different components for aggregate total factor productivity. Contrary to our static measures, the effects of ex ante misallocation are by far the most profound: TFP is about 40% lower compared to that of the benchmark economy without adjustment costs, revenue distortions and information frictions. Moreover, also uncertainty has a substantial effect on TFP, reducing it by around 8-12%.[9] Finally, the effects of adjustment costs depend on whether they are evaluated alone or in conjunction with other components. In the former case, TFP is reduced by 5%, while in the latter case, the effects are minuscule.

2 Related Literature

Our approach of evaluating static misallocation together with uncertainty is related to a branch of new research that relaxes the original assumptions of HK in ways that allow for dispersion in TFPR to also reflect other things besides misallocation. Haltiwanger et al. (2018) consider more general demand and production structures. Bils et al. (2021) and Rotemberg and White (2021) take into account the possibility of measurement errors, while Gollin and Udry (2020) develop a framework that allows them to separate between measurement error, unobserved heterogeneity and misallocation. Baqaee and Farhi (2020) allow for flexible input-output linkages and varying substitutability in a non-parametric framework. Eslava et al (2023) utilize price and quantity data and decompose misallocation wedges into components arising from input prices, markups and residual terms. Related to us, they also observe that misallocation wedges decrease as firms age. In relation to this literature, we show that uncertainty, especially in the early life-cycle, can explain a large fraction of the dispersion in TFPR.

[9] Uncertainty alone reduces TFP by 12%. When uncertainty is added to a setup where we already have revenue wedges and adjustment costs, the reduction is 8%

In trying to quantify the effects of misallocation researchers have also taken an alternative approach, where the role of specific source(s) of "misallocation", variation in marginal product(s) of labor and/or capital, is tried to capture with the help of a structural model. Midrigan and Xu (2014), for example, examine the role of financial markets, while Asker et al. (2014) analyze the role of adjustment costs. Bartelsman et al. (2013) study misallocation when capital is quasi-fixed and firms' use overhead labor.

Within the branch of studies that utilize the more structural approach, a few papers evaluate the role of information frictions. In David et al (2016), as in our setup, firms choose their inputs (or part of them) before they know their fundamentals. Firms also learn from stock markets in their framework. In our data, a large majority of firms is unlisted, thus, we do not allow this type of learning. Another paper exploring the role of uncertainty with the help of a structural model is David and Venkateswaran (2019). They develop a tractable framework that allows them to measure several sources of capital misallocation, such as adjustment costs and tax-like wedges, jointly with uncertainty. These papers, like most of the misallocation literature, do not consider life-cycle aspects and thus, the uncertainty is revealed at the end of each period. Moreover, they do not allow for endogenous selection.[10] Regarding life-cycle aspects and selection, a notable exception is Feng (2022), who observes that misallocation is decreasing in Chinese firms and also considers Jovanovic-style learning an explanation for this.[11] We view our semi-structural accounting setup as complementary to this literature. It allows one to explore data patterns related to uncertainty and misallocation and, thus, is potentially useful in guiding the modeling choices and evaluating the model's ability to replicate the observed patterns.

Our paper is also connected to a branch of literature that aims to

[10]This type of interaction could be important for the aggregate effects of micro-level misallocation, as illustrated by, e.g., Yang (2021).

[11]Along these dimensions, the setup of Tian (2022) is close to ours. In her model, firms are always uncertain about the quality of their product and the state of the economy. In contrast to the papers mentioned earlier, her goal is not to evaluate the quantitative importance of uncertainty as a channel of misallocation, but to analyze the connections between the two sources of uncertainty more broadly.

understand the life-cycle growth of firms. In Clementi and Plazzo (2016), adjustment costs generate age-dependent growth and exit rates. Sterk et al. (2021) use a model with monopolistic competition, similar to ours, to understand the determinants of the up-or-out-dynamics that characterize the growth of new businesses. Their results emphasize ex ante heterogeneity. Within this tradition, a paper closest to ours is Arkolakis et al. (2018). They illustrate that the Jovanovic (1982) style learning combined with a GE framework can generate life-cycle profiles that are in line with the US data.

A few recent papers emphasize the connections between productivity investments and misallocation. Hsieh and Klenow (2014) allow the firms' TFP to evolve endogenously over the life-cycle.[12] In their approach, misallocation can severely discourage investments on the productivity and thus dampen the aggregate TFP. In Bento and Restuccia (2017), firms can invest on productivity not just along the life-cycle but also upon entry. Peters (2020) considers the effects of market power in generating misallocation. In his setup, the market power of firms is endogenous and evolves over the life-cycle.

3 Accounting for Uncertainty and Misallocation

In this section, we develop an accounting framework to jointly measure misallocation and uncertainty in a static setup with a minimum amount of theory by just specifying a production function and a demand structure. We link our measure to the dispersion of the HK revenue wedge, a standard measure of misallocation when there are no distortions that affect capital and labor asymmetrically. Next, we apply our approach to Finnish firm-level data and show that uncertainty makes up for around 40% of ex post misallocation. We also find that there is a strong age-dependent trend in uncertainty. Finally, we explore the robustness of our results.

[12] They build on Atkeson and Burstein's (2010) GE model. The approach is close to papers that examine the role of investments in organizational capital and customer base. See, e.g., Foster et al. (2016)

3.1 A Theoretical Accounting Framework

This subsection develops a simple way to indirectly measure the wedges that uncertainty and tax-like revenue distortions can generate in static first order conditions. Firms face CES demand and produce with Cobb-Douglas technology using labor and capital. They choose their inputs under imperfect information about the current period fundamentals. To keep the exposition of the framework as simple as possible, we only focus on firms' uncertainty over their productivity. However, since we do not have data on quantities, this interpretation is observationally equivalent to assuming that demand is also uncertain. In addition, firms face idiosyncratic revenue distortions that are known to the firms at the time they make their production decisions.

There is a large number of firms, indexed by i, each of them producing a differentiated good. Individual goods are aggregated to a single final good with the CES aggregator. Thus, firm i in industry s at time t faces the isoelastic demand curve given by

$$y_{t,s,i} = \left(\frac{p_{t,s,i}}{P_t}\right)^{-\sigma} Y_t, \tag{1}$$

where $p_{t,s,i}$ is the price of good, P_t is the price index and Y_t is the the amount of final good consumed.

The production technology for each firm is represented by a Cobb-Douglas production function of a firm's TFP, $z_{t,s,i}$, labor, $n_{t,s,i}$, and capital, $k_{t,s,i}$.

$$y_{t,s,i} = z_{t,s,i} n_{t,s,i}^{1-\alpha_s} k_{t,s,i}^{\alpha_s}, \tag{2}$$

where the capital intensity, α_s, is allowed to vary across industries. In the empirical section we also consider a specification of the production function where the capital intensity is allowed to vary across firms and time.

Given the demand structure and the production function, the objective of firm i is to maximize expected profits given by

$$E(\pi_{t,s,i}) = Y_t^{\frac{1}{\sigma}} P_t (1-\tau_{t,s,i}) E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}}) (n_{t,s,i}^{1-\alpha_s} k_{t,s,i}^{\alpha_s})^{\frac{\sigma-1}{\sigma}} - w_{t,s,i} n_{t,s,i} - R_t k_{t,s,i}, \tag{3}$$

where $\tau_{t,s,i}$ is a tax-like idiosyncratic distortion that firm i faces in period t. We allow wage rates, $w_{t,s,i}$, to be specific to each firm and time period because we use the wage bill as an observable variable in the empirical part of the paper. This approach enables the observed variation in wage bills to arise from either employment levels or wage rates. On the other hand, the rental rate, R_t, is assumed to be common to all firms in our baseline calculations in Section 3.2. In Subsection 3.3, we relax this assumption and use firm-specific implicit interest rates as proxy for firm-time-specific rental rates.

From the first order condition with respect to labor, we get the following expression for the distortion:

$$1 - \tau_{t,s,i} = (1 - \varphi_{t,s,i})^{-1} \frac{\sigma}{(\sigma - 1)(1 - \alpha_s)\frac{p_{t,s,i}y_{t,s,i}}{w_{t,s,i}n_{t,s,i}}}, \tag{4}$$

where $1 - \varphi_{t,s,i}$ measures the prediction error of the firm,

$$1 - \varphi_{t,s,i} \equiv \frac{E(z_{t,s,i}^{\frac{\sigma-1}{\sigma}})}{z_{t,s,i}^{\frac{\sigma-1}{\sigma}}}.$$

To be able to write this equation in terms of observable valued added, $p_{t,s,i}y_{t,s,i}$, we have multiplied and divided the first order condition with $z_{t,s,i}^{\frac{\sigma-1}{\sigma}}$.

Equation (4) emphasizes the fact that without knowledge of the firm's information set at the time when inputs are chosen, the observed ratio between realized value added, $p_{t,s,i}y_{t,s,i}$, and wage stock, $w_{t,s,i}n_{t,s,i}$, is not enough to identify tax-like distortion, $1-\tau_{t,s,i}$. Moreover, given that capital is also chosen under uncertainty, the first order condition with respect to it does not help us to separate between the prediction error and the revenue tax. To solve this identification problem, we use additional information embedded in realized profits.

We can highlight information in profits by writing the realized profits with the help of optimal policies for labor and capital:

$$\pi_{t,s,i} = (1 - \tau_{t,s,i})p_{t,s,i}y_{t,s,i}(1 - \frac{\sigma - 1}{\sigma}(1 - \varphi_{t,s,i})) \tag{5}$$

Ideally, firms would like to have a constant ratio of profits to value added. It is also interesting to note that with tax-like frictions but without prediction errors, this ratio would give an additional condition from which to identify $1-\tau_{t,s,i}$.

With both information frictions and tax-like distortions, it turns out that the ratio of profits to wage stock allows us to pin down prediction error, $1-\varphi_{t,s,i}$. To see this, note that eq (4) implies that

$$w_{t,s,i}n_{t,s,i} = (1-\alpha)(1-\tau_{t,s,i})(1-\varphi_{t,s,i})\frac{\sigma-1}{\sigma}p_{t,s,i}y_{t,s,i}. \tag{6}$$

Dividing eq (5) with eq (6) and solving for $1-\varphi_{t,s,i}$ gives

$$1-\varphi_{t,s,i} = \frac{\sigma}{\sigma-1}\left(\frac{1}{1+(1-\alpha_s)\frac{\pi_{t,s,i}}{w_{t,s,i}n_{t,s,i}}}\right) \tag{7}$$

Both the wage stock and realized profits depend on $(1-\tau_{t,s,i})$ in a similar way, thus the ratio of profits to wage stock is independent of the distortion responsible for ex ante misallocation. The effects of $1-\varphi_{t,s,i}$, however, are asymmetric for $\pi_{t,s,i}$ and $w_{t,s,i}n_{t,s,i}$. Essentially, this happens because the optimal employment is determined by the expected fundamentals, while the profits depend on the realized fundamentals as well as the expectations.[13]

Next, we can solve $1-\tau_{t,i,s}$ from eq (4). Taken together, equations (4) and (7) state that we can measure firm-(time-)specific distortions $(1-\tau_{t,s,i})$ and prediction errors $(1-\varphi_{t,s,t})$ if we observe two key ratios: a firm's value added to wage stock and profits to wage stock.

The relationship between our measure of misallocation distortion and the one presented in HK becomes clear when the expression of our distortion, equation (4), is substituted into the equation that defines the firm's profits:

$$\pi = Y_t^{\frac{1}{\sigma}}P_t(1-\tau_{t,s,i}^{HK})z_{t,s,i}^{\frac{\sigma-1}{\sigma}}(n_{t,s,i}^{1-\alpha_s}k_{t,s,i}^{\alpha_s})^{\frac{\sigma-1}{\sigma}} - w_{t,s,i}n_{t,s,i} - R_tk_{t,s,i}, \tag{8}$$

[13]The realized fundamental, $z_{t,s,i}^{\frac{\sigma-1}{\sigma}}$ enters the wage stock equation only because we have written it in terms of actual valued added.

where

$$1 - \tau_{t,s,i}^{HK} = \frac{\sigma}{(\sigma - 1)(1 - \alpha_s)\frac{p_{t,s,i}y_{t,s,i}}{w_{t,s,i}n_{t,s,i}}} \tag{9}$$

is the revenue distortion in HK. Thus, if one ignores uncertainty, ex post it looks like firms were maximizing (8). Equations (9) and (4) also make it clear that our approach decomposes the standard measure of revenue distortion,

$$\ln(1 - \tau_{t,s,i}^{HK}) = \ln(1 - \varphi_{t,s,i}) + \ln(1 - \tau_{t,s,i}), \tag{10}$$

to a component reflecting ex ante distortions $(1 - \tau_{t,s,i})$ and prediction error $(1 - \varphi_{t,s,i})$. For this reason, we also call $\tau_{t,s,i}$ a residual wedge.

Without capital frictions, variation in the log of HK revenue distortion also gives the variation in the log of TFPR, which is the standard indirect measure of misallocation.[14] We can use the decomposition given by (10) to rewrite the misallocation measure as

$$\mathrm{Var}(\ln(1-\tau^{HK})) = \mathrm{Var}(\ln(1-\varphi)) + \mathrm{Var}(\ln(1-\tau)) + 2\,\mathrm{Cov}(\ln(1-\varphi), \ln(1-\tau)). \tag{11}$$

That is, in the presence of uncertainty, the measure of ex post misallocation can be decomposed to components reflecting uncertainty (the variance of prediction error), ex ante misallocation (the variance of tax-like distortion) and the covariance between the two.

3.2 Measuring Misallocation in Finnish Data

We utilize the framework introduced in the previous subsection to analyze the ex post misallocation and its decomposition for Finnish firms. Moreover, we also explore the age-dependent trends in these measures.

We use annual firm-level data from the Financial Statement Statistics for years 1989–2012, provided by Statistics Finland. However to minimize measurement errors, we focus on years 1995–2012, for which Statistics Finland utilizes the tax register data of businesses as their primary source of financial

[14] See HK for details.

statement data. For this period, our data covers the vast majority of Finnish firms across industries, excluding the financial sector. The coverage varies between 95% and 99% of all Finnish firms. Earlier, during the years 1989–1994 Statistics Finland relied on their own survey with a substantially lower coverage of firms.[15] Thus, we only use the period of 1989-1994 in order to determine the firms' age. For this purpose we also use the Business Register data on the establishment level for years 1989–2012. This data is also from Statistics Finland.

We focus on industries 15-63 with NACE rev 2 codes. That is, in addition to finance, insurance and real estate, we also omit agriculture and mining industries. To focus on firms with meaningful balance sheet information in which the work effort of the owners is not the only source of labor input, we only report results for limited liability companies who on average have more than one worker. As an additional restriction, we only follow firms up to the age of 10 years when we examine age-dependent trends. In theory, the data would allow us to follow some firms up to the age of 24. However, the number of firms for older generations is so low that we omit these cohorts when we do our accounting exercise conditional on the firms' age.

The variables we use are value added, employment compensation (wages and salaries plus other personnel expenses), total profits, equity and industry code at a three digit level. In our static model, firms rent capital. To get the measure of profits in line with this, we deduce the opportunity cost of a firm's own capital, 5% real interest rate times firms total equity, from its total profits.[16] A firm's age is determined based on the year in which the first establishment appears in the business register data. Appendix A presents some summary statistics for these variables and the key ratios that allow us to pin down different wedges.

We follow the baseline HK setup and attach an industry-specific, time-

[15]Statistics Finland have also retrospectively added the administrative data for 1994. However, the coverage in 1994 is not comparable to the subsequent period.

[16]We have also performed our baseline calculations using firm-specific implicit interest rates calculated by dividing a firm's interest payments with its borrowed capital. These results are reported in Subsection 3.3.

constant capital elasticity, α_s, to each firm using the labor shares at the 3-digit level industries. Later, in Section 3.3, we also consider firm-time-specific capital elasticities, $\alpha_{t,s,i}$. Finally, we assume a common σ for all firms. Given that our focus is on the variances of $\ln(1-\tau)$ and $\ln(1-\varphi)$, and the covariance between the two components, the exact value of σ does not affect our results. We have also conducted our calculations using industry-specific fixed effects to account for variations in demand elasticities across industries. This adjustment does not affect our results.

We start by calculating the HK revenue wedge and its components using equations (4), (7) and (9) for all limited liability companies with more than one worker on industries 15-63 for years 1995-2012. To increase the reliability of our misallocation measures, we winsorize the resulting wedges at 0.01-level. Next, we calculate the variance of (log) HK revenue wedge, the variance of its components and their covariance.

Table 1: The variance of HK style revenue wedge and its decomposition to uncertainty and residual wedge.

Variable	Value	Share
$\text{Var}(\ln(1-\tau_{HK}))$	0.207	1.00
$\text{Var}(\ln(1-\tau))$	0.075	0.36
$\text{Var}(\ln(1-\varphi))$	0.084	0.41
$2\text{Cov}(\ln(1-\tau), \ln(1-\varphi))$	0.045	0.22

Notes: Percentages do not necessarily sum up to one because of the winsorization.

The results are given in Table 1. The variance of log HK wedge is 0.207. Utilizing the simple closed-form expression for the aggregate TFP losses from HK with the assumption of joint log-normal distribution for productivity and TFPR, we get an idea of the aggregate significance of the ex post misallocation.[17] If we assume that σ is 3, the TFP gains from eliminating the variation in the HK wedge would be 31%. The observed ex post misallocation is

[17] See eq (16) in HK

in line with the HK numbers for the US manufacturing plants. This is perhaps slightly surprising, given that the data sets used are not directly comparable. Our unit of observation is firm while they use plant-level data. Moreover, our data also contains a majority of small firms for which the distortions are likely to be more severe. Note also that unlike many others, we did not restrict our sample to manufacturing firms. When only exploring the manufacturing sector, the dispersion of the revenue wedge is slightly smaller (see Appendix B).

Moving on to the variance decomposition, we can see that uncertainty accounts for 41% of the total variation. The variance of the residual wedge, our indirect measure of misallocation, makes up 36% of the variation in the HK style misallocation measure. Using the back of the envelope calculation that relies on the joint log-normality and ignoring the covariance term, these numbers turn into TFP gains of 13% and 11%, respectively. Thus, it seems that uncertainty and idiosyncratic revenue distortions are both important in generating ex post misallocation and TFP losses.

Finally, in the pooled data the covariance of prediction error and revenue wedge is positive and it has a non-negligible role in accounting for the ex post misallocation. Taken at face value, positive covariance would suggest that firms with high $1 - \tau$ are over-optimistic about their productivity. Alternatively, this could also be caused by omitted heterogeneity. This point is illustrated in row c of Table 3, where we allow for interest rate heterogeneity between firms.

In appendixes, we decompose the results of the pooled data. In Appendix B, we discuss the results of our decomposition over different industries. The main observation from this exercise is that the results are remarkably stable across industries. Uncertainty and ex ante misallocation are more or less equally important, both accounting for around 40% of the total variation. The relative importance of the covariance term is typically around 20%. The only industry that stands out is electricity, gas and water supply with a higher ex ante misallocation. This is perhaps to be expected, given the tight regulation and limited competition in that industry. In Appendix C, we report the decomposition for a set of individual years. Again, our results are close to those of the pooled data. The only exception is that uncertainty is somewhat elevated in the first two years of our sample.

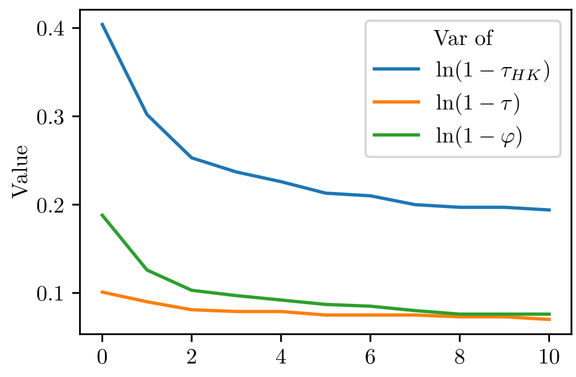
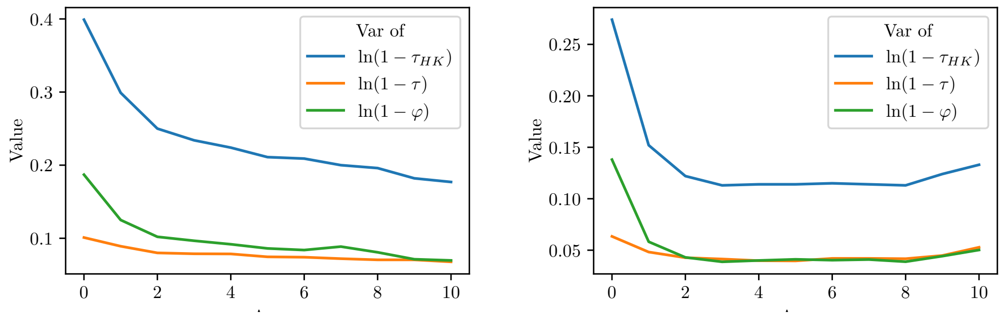


Figure 1: Uncertainty and misallocation for Finnish firms conditional on firms' age.

Finally, we explore the life-cycle aspects of misallocation. We calculate the variance of the HK revenue wedge conditional on firms' age. We also redo our decomposition separately for all age groups and report the variances of prediction errors and the residual wedges. The results of this exercise are reported in Figure 1. As stated earlier, there is a strong negative trend in ex post misallocation (the blue line). The variance is almost twice as high for entrants than for firms that are ten years old. The majority of this trend is accounted for by decreasing uncertainty (the green line), which is more than halved. Contrary to these patterns, the orange line that gives the dispersion in the residual wedge is practically constant after an initial small drop that takes place during the first two years.

As young firms also tend to be small, it is useful to investigate, whether reducing uncertainty is, in fact, driven by increasing production size instead of ageing. This could happen, for example, if productivity (and/or demand) shocks tended to wash out more for larger firms. We evaluate this channel by first running a regression where we explain variables $1-\tau_{HK}$, $1-\tau$ and



(a) Size controlled (b) "Balanced panel"

Figure 2: Uncertainty and misallocation conditional on firms' age. The left panel gives the result after controlling for the size of the firm, while the right panel gives result for a "balanced panel" that only contains the firms that survive as a minimum up to their tenth year.

$1 - \varphi$ with firm size. Afterwards, we take the residuals of these regressions and report their variances conditional on age. The results are given in the left panel of Figure 2. Comparing it with Figure 1, we see that the age-dependent trends are practically unaltered, thus the firm size has a negligible effect on the results. In Appendix C, we report the life-cycle patters of misallocation separately for different cohorts. It illustrates that the observed patterns are not driven by a single cohort.

On the right panel of Figure 2, we report the age-related accounting exercise for a "balanced panel", a subset of firms that survive at least up to tenth year. Given our sample, these are firms that were founded between 1995-2002. From the figure, we see that uncertainty and ex post misallocation also reduce with age for the group of survivors. Again, the reduction in uncertainty is substantial. Comparing this figure with Figure 1, however, reveals that the convergence of uncertainty is faster for the group of survivors.

Motivated by the differences between Figure 1 and the right panel of Figure 2, we explore the association between prediction errors made by firms and their exit in the next period.[18] The results of a linear probability model (LMP) and

[18]Given the administrative nature of our data, firms are highly unlikely to disappear from our data unless they have truly stopped existing as independent firms. Unfortunately,

a logit model based on our whole dataset are assembled in Table 2. The first two columns show the results when we only use the prediction error from the previous year, in addition to year and industry fixed effects, as an explanatory variable. As we can see, there is a statistically significant positive association between exits and prediction errors. For example, according to the linear probability model, one standard deviation (0.29) increase in prediction error leads to an increase of 0.6% in exit probability. Given that the unconditional exit probability in our data is 3.1%, this is a substantial increase. The third and fourth columns include the prediction error from two periods before as an additional regressor. These columns highlight the persistence of the relationship between exits and prediction errors.

The age decomposition hints that reducing uncertainty about fundamentals could play a significant role in understanding the life-cycle patterns of resource allocation. In addition, the decreasing trend in uncertainty, together with a positive association between prediction errors and exits, is in line with the Jovanovic (1982) style mechanism of firm growth, where learning plays an important role in explaining the observed up-or-out-style life-cycle patterns. In the next subsection, we explore some alternative explanations. We also give further evidence of the learning at the firm level by considering age dependency after controlling for firm-specific fixed effects.

however, we are not able to separate between closures, mergers and acquisitions.

Table 2: Exits and prediction errors

	Dependent variable: Exit at the Beginning of Next Period			
	LPM	Logit	LPM	Logit
$\ln(1-\varphi)$	0.021	0.670	0.017	0.619
	(0.001)	(0.018)	(0.001)	(0.022)
$L(\ln(1-\varphi))$			0.018	0.682
			(0.001)	(0.022)
APE $\ln(1-\varphi)$		0.018		0.016
APE $L(\ln(1-\varphi))$				0.018
Industry FE	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
N	722721	722721	623072	623072

Notes: For the linear probability models (LPMs), we report robust standard errors. L() refers to lagged value.

3.3 Sensitivity of the Results for Alternative Specifications

Our indirect approach suggests a substantial role for uncertainty in generating ex post revenue misallocation and decreasing the age-dependent trend in it. In this subsection, we explore the robustness of these results for different forces that might generate variation in the observed profits-to-wage stock and value-added-to-wage stock ratios. We start by allowing production heterogeneity across firms, within industries and across time. Next, we relax the assumption of homogeneous interest rates. Then, we give up the assumption that capital needs to be freely adjustable. Finally, we utilize the panel dimension of our data by introducing firm-specific fixed effects. This enables us to control for differences in unobserved firm heterogeneity. We also use the firm fixed effect setup to provide some indicative evidence that improvements in forecasting precision also happen within firms as they age. The results of these exercises are reported in Table 3 and Figure 3. At the end of this subsection, we shortly discuss the results related to an extended setup where we also allow for labor distortions (see Appendix D for details).

In the previous subsection, we used industry-specific capital elasticities. However, as David and Venkateswaran (2019) have highlighted, heterogeneity in production technologies, in the form of varying capital intensities, can potentially be important for observed misallocation. To take this into account, we consider firm-time-specific capital elasticities, $\alpha_{t,s,i}$, by identifying them from capital-labor-ratios at time t at the firm-level. The first order conditions of our firm's problem imply that

$$\frac{1-\alpha_{t,s,i}}{\alpha_{t,s,i}} = \frac{w_{t,s,i} n_{t,s,i}}{R_t k_{t,s,i}}.$$

Note that in the presence of relative distortions that affect capital and labor asymmetrically, this approach exaggerates the variation in the capital elasticity. However, together with the alternative extreme, a common industry-specific, α_s, this gives us information about the sensitivity of our results to production heterogeneity.

Table 3: Sensitivity of static misallocation and its components

	$\mathrm{Var}(\ln(1-\tau_{HK}))$	$\mathrm{Var}(\ln(1-\tau))$	$\mathrm{Var}(\ln(1-\varphi))$	$2\mathrm{Cov}(\ln(1-\tau),\ln(1-\varphi))$
a) Baseline	0.207	0.075 [36%]	0.084 [41%]	0.045 [22%]
b) Firm-specific α	0.158	0.072 [46%]	0.059 [37%]	0.019 [12%]
c) Het. interest rate	0.207	0.100 [48%]	0.090 [43%]	0.002 [1%]
d) Pot. inadjustable capital	0.207	0.072 [35%]	0.173 [84%]	-0.039 [-19%]
e) Fixed effects	0.095	0.033 [34%]	0.048 [51%]	0.012 [13%]

Notes: Relative contributions are given in brackets. These percentages do not necessarily sum up to one because of the winsorization.

The results with production heterogeneity are shown in row b of Table 3. Row a of the table reproduces the baseline results from the previous subsection. A comparison of these rows reveals that allowing for production heterogeneity

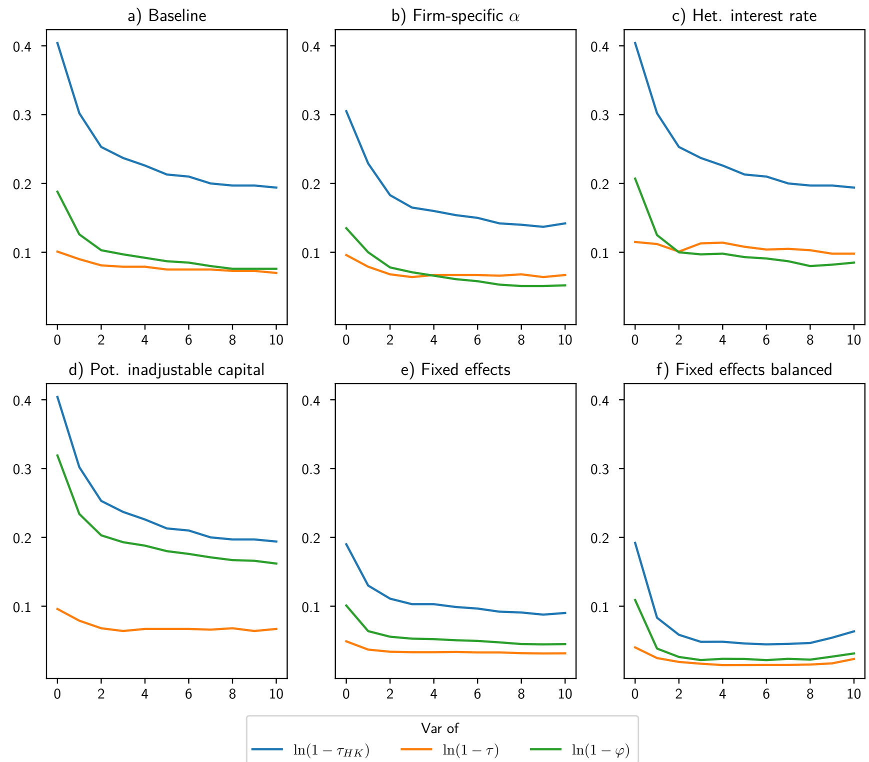


Figure 3: Age patterns for alternative specifications.

reduces ex post misallocation by 24%, from 0.207 to 0.158. Next, looking at the decomposition, we see that the covariance term is more than halved when heterogeneity is allowed. In line with this, its relative contribution to ex post misallocation (reported in brackets) is also substantially reduced. Another change is that the role of uncertainty is lower now, though its relative importance is still close to 40%. Unlike for the other components, there is no substantial drop in ex ante misallocation. This, together with the reduction of other components, implies that the relative contribution of ex ante misallocation is now somewhat higher.

In panel a of Figure 3, we again reproduce the baseline life-cycle aspects of misallocation and its components, while in panel b we use firm-time-specific elasticities. Comparing the two figures reveals that even though the starting values are lower with production heterogeneity, the general patterns are similar: ex post misallocation is halved during the first ten years, as is the uncertainty component, while ex ante misallocation is stable after the first two years in both cases.

A branch of recent literature has highlighted the substantial variation in interest rate spreads and its implications for aggregate outcomes (see e.g., Cavalcanti et al, 2021, Gilchrist et al, 2013 or Bai et al, 2018). Even though we focus on symmetric wedges, this variation could still be relevant for our decomposition since it could alter the observed ratios of monopoly profits to wage stock. Up to this point, when generating our profit measure, we have assumed that the rental rates of capital are equal across firms. However, a heterogeneous default risk, for instance, could mean that the required return to a firm's own capital also varies between firms. We explore the importance of this channel by using firm-time-specific implicit interest rates as a proxy for returns required by the own capital. That is, we take the profit measure from financial statement data and subtract the firm's own capital multiplied by a firm-time-specific interest rate from it. When calculating these interest rates, we divide interest rate payments by debt.

The firms' borrowing costs are also heterogeneous in Finland; the standard deviation of implicit interest rates is 2.5% (with the mean rate being 2.1%). However, when we recalculate our decomposition using the profit measure that takes this into account, our decomposition results are only slightly altered. From row c of Table 3, it can be seen that the role of ex ante wedges is somewhat fostered, as ex ante misallocation is now generating almost 50% of the total variation. The relative contribution of uncertainty is 43%. The most significant change compared to the baseline relates to the level of covariance term which is now practically zero. Panel c of Figure 3 reports the evolution of misallocation and its components conditional on firms' age with heterogeneous interest rates. The profile of ex ante misallocation is in line with the previous results, though there is also a little variation after the first two years. As to uncertainty, the starting values are somewhat higher than in the benchmark case, while the reduction in the first years is a bit faster.

In our baseline static framework, both labor and capital were assumed to be variable inputs that could be freely chosen in each period. However, for example, adjustment costs (e.g, Asker et al, 2014) or credit rationing (e.g., Buera et al, 2011) could restrict the feasible levels of capital, as well as imply that capital is a dynamic production input. To examine how these types of constraints might affect our results, we proceed by being agnostic about how the level of capital is determined and only assume that labor is a variable input of production.

Conditional on the level of capital, the first order condition with respect to labor is still given by equation (4). Defining a new measure of profits as monopoly profits plus rental rates on capital, $\pi^*_{t,s,i} \equiv \pi_{t,s,i} + R_t k_{t,s,i}$, we have

$$\pi^*_{t,s,i} = (1-\tau_{t,s,i})p_{t,s,i}y_{t,s,i} - w_{t,s,i}n_{t,s,i}$$
$$\pi^*_{t,s,i} = (1-\tau_{t,s,i})p_{t,s,i}y_{t,s,i}(1 - \frac{\sigma-1}{\sigma}(1-\alpha)(1-\varphi_{t,s,i})).$$

Dividing this expression with wage stock, $w_{t,s,i}n_{t,s,i}$, and solving for $1-\varphi_{t,s,i}$ gives

$$1-\varphi_{t,s,i} = \frac{\sigma}{\sigma-1}\frac{1}{1-\alpha}\frac{1}{1+\frac{\pi^*_{t,s,i}}{w_{t,s,i}n_{t,s,i}}}. \tag{12}$$

We can again insert this expression into eq (4) to solve the ex ante wedge, $1-\tau_{t,s,i}$.

Row d of Table 3 assembles the results of our indirect approach when we do not take a stance on how the level of capital is chosen. Given that the value-added-to-wage-stock ratios are not altered, the measure of ex post misallocation is not changed. The most notable difference compared to the previous results is the elevated level of uncertainty. Another difference is that the sign of the covariance term is now negative. The level of ex ante misallocation, on the other hand, is unaltered. Panel d of Figure 3 gives the life-cycle profiles for this case. From it, we see that uncertainty goes up for all ages and closely tracts the profile of ex post misallocation. The age pattern of ex ante misallocation again resembles that of the baseline results.

We also utilize the panel dimension of our data to control for unobserved firm heterogeneity. To do this, we take the (log) wedges generated by our baseline approach and run regressions where we use these wedges as dependent variables and firm fixed effects as explanatory variables. We take the residuals of these regressions, given that a firm has at least two observations, and report the variation in the residuals. This approach allows us to control for, for example, heterogeneity on markups or fixed costs. On the downside, we are also likely to throw away relevant variation in the data. For uncertainty, this happens, for example, if due to learning, firms' prediction errors are persistent and firms that are over-optimistic about their productivity exit after observing a few bad signals in a row. Thus, it seems likely that the fixed-effect setup gives down-ward biased results ex post misallocation and its components.

The results of the fixed effect setup are shown in row e of Table 3. As expected, compared to the benchmark results, the variation in ex post wedge is now reduced. The same is true for all of its components. Interestingly, the drop in relative terms is smaller for uncertainty than it is for ex ante misallocation and the covariance term. Despite these somewhat uneven reductions, the relative importance of uncertainty and ex ante misallocation is still in line with the previous results. Panel e of Figure 3 illustrates the age-related patterns with fixed effects. The broad patterns in ex post misallocation and uncertainty are similar to the baseline results though with reduced level effects. To illustrate role of learning within firms vs selection, we repeat the fixed effect graph with with "balanced panel", i.e., for firms that survive up to the tenth year in panel f of the figure. From this we see that the uncertainty is reducing for surviving firms also after controlling for unobserved firm heterogeneity.

Finally, in Appendix D we extend our approach to include labor distortions. To do this, we consider a more general setting, where firms also choose their use of materials. In order to separate ex ante wedges from prediction errors, we assume that firms select their materials after they know their productivity. When prediction error does not correlate with output wedge, this approach potentially underplays the role of uncertainty and overplays the role of ex ante wedges if firms also face some uncertainty regarding their use of materials.

The results of our extended accounting exercise indicate (see Table 11 and

Figure 7 in Appendix D) that the role of uncertainty is, again, about 40% of the variation in the HK wedge. Moreover, uncertainty still has a clear age-dependent downward trend. However, there is a substantial upward shift in ex ante misallocation for all age groups. Given that ex ante misallocation now reflects not just the role of output wedges, but also the contribution of labor wedges, this result could be seen as a sign of the importance of labor distortions in Finland. For example, a highly centralized wage setting with a strong union coverage together with rigid labor protection practices could be generating substantial misallocation.

4 Model

Motivated by the results from our static decomposition of misallocation, we set up a general equilibrium model, where firms have to choose their input without full information about the current period productivity. To allow for the observed age-dependent trend in the prediction errors, we add Jovanovic's (1982) learning mechanism to a general equilibrium framework, similar to Hopenhayn and Rogerson (1993) and Melitz (2003).

4.1 Households

There is a unit mass of risk neutral infinitely lived households which derive utility from consumption and supply labor inelastically. The behavior of households can be summarized with a representative household, whose preferences are given by $\sum_{t=0}^{\infty} \beta^t C_t$, where C_t is a consumption basket compiled of individual goods with the CES aggregator such that

$$C_t = \left(\int_{\Omega_t} c_{i,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{13}$$

where Ω_t is the amount of goods available. The household owns the firms and thus the budget constraint is given by

$$\int_{\Omega_t} p_{i,t} c_{i,t} = w_{t,s,i} \bar{N} + \Pi_t, \tag{14}$$

where Π_t are aggregate profits and $w_{t,s,i}$ is the wage rate. We focus on stationary equilibrium and, thus, from now on, we drop the time indexes. To ease the notation, we also drop the firm index i. Moreover, we use labor as the numéraire.

4.2 Incumbent

There is an endogenous measure of incumbent firms denoted by Ω. Each firm produces a unique good and faces a demand in line with (13) and (14),

$$y = \left(\frac{p}{P}\right)^{-\sigma} C. \tag{15}$$

The production function of a firm is given by linear technology

$$y = e^z n, \tag{16}$$

where e^z is the firm's TFP and n the employees hired by the firm. The firm-specific TFP in the current period is given in logs as

$$z = z_p + z_t. \tag{17}$$

In the equation, z_p is a persistent productivity component that for each firm follows an AR(1) process:

$$z_p = \rho z_{p,-} + \epsilon_p, \tag{18}$$

where $0 < \rho < 1$ and $z_{p,-}$ is the value of the persistent component in the previous period. The innovation term, ϵ_p, is distributed according to $\epsilon_p \sim N(0, \sigma^2_{\epsilon_p})$. For a new firm, the initial persistent component is drawn from a normal distribution with mean zero and variance $\frac{\sigma^2_{\epsilon_p}}{1-\rho^2}$. Finally, the other component in equation (17), z_t, is temporary productivity, which is drawn from a normal distribution in each period. We assume that $z_{i,t} \sim N(0, \sigma_{z_t}^2)$.

A firm observes z but is unable to decompose it. In line with the analysis in the previous section, we assume that the firm needs to choose its employment before it observes z. This implies that in each period, t, the firm needs to form an estimate of the persistent productivity component conditional on the history of observed zs up to $t-1$. We denote this prediction with m. As more information is accumulated, m becomes more precise.

The firm uses Bayesian learning to update its expectation. Given the log-normality, we get the standard Kalman filter with the following recursive representation for the prediction, m, and the variance of the firm's expectation of its permanent productivity, Σ,

$$m' = \rho m + K(z - m) \tag{19}$$

$$K = \frac{\rho \Sigma}{\Sigma + \sigma_{z_t}^2} \tag{20}$$

$$\Sigma' = \frac{\rho^2 \Sigma \sigma_{z_t}^2}{\Sigma + \sigma_{z_t}^2} + \sigma_{\epsilon_p}^2 \tag{21}$$

(See, e.g., Ljungqvist and Sargent, 2018, for details). We assume that all firms start with common priors, i.e., with the unconditional persistent productivity distribution:

$$m_0 = 0, \quad \Sigma_0 = \frac{\sigma_{\epsilon_p}^2}{1 - \rho^2}.$$

Given this structure, the firm's expected value for m' before observing productivity z is distributed as

$$E(m' \mid m, \Sigma) \sim N(\rho m, \rho \Sigma K).$$

In addition, the distribution of z conditional on productivities up to $t-1$ is given as

$$z = m + (z_p - m) + z_t \sim N(m, \Sigma + \sigma_{z_t}^2). \tag{22}$$

Following, e.g., Restuccia and Rogerson (2008), we do not explicitly model the sources of misallocation, but assume that these frictions can be summarized by an idiosyncratic distortion, $1 - \tau$, that appears in the firm's profit maximization problem as a revenue tax would. For simplicity, we assume that the firm-specific distortion only contains a permanent component. That is,

$$\ln(1 - \tau) = \tau_p, \tag{23}$$

where τ_p is normally distributed and drawn upon entry. In addition, operating firms also have to pay periodic fixed costs, c_f.

Given the demand structure, the productivity process and the revenue wedge, a firm's objective is to maximize its lifetime profits by making an optimal exit/stay decision and conditional on stay to choose the current period employment. As stated earlier, we assume that this decision is made before the current period productivity is known. Thus, the firms' marginal productivities will differ due to idiosyncratic revenue wedge and expectation error. In addition to these channels, we also allow for convex adjustment costs that could also be a potential explanation for the age-dependent trend in misallocation.

The intra-period timing is summarized in Figure 4. At the beginning of a period, an incumbent firm chooses whether it wants to exit or not. A firm that decides to continue, pays periodic fixed costs, c_f. Next, it chooses its employment. If a continuing firm decides to adjust its scale, the firm has to pay an adjustment cost, $\lambda(\frac{n-n_-}{\bar{n}})^2\bar{n}$, where $\bar{n} = \frac{n+n_-}{2}$. After choosing its employment level, the firm produces and observes the current period (combined) productivity. Finally, at the end of the period, the firm may be forced to exit due to an exogenous shock that happens with probability γ.

Taken altogether, the firm's problem at the beginning of a period can be summarized by the following recursive expression:

$$V(m, a, n_-, \tau_p) = \max\{W(m, a, n_-, \tau_p), -2\lambda n_-\}, \tag{24}$$

where the relevant state variables are the firm's belief about its permanent productivity, m, firm's age, a, its employment in the previous period, n_-, and

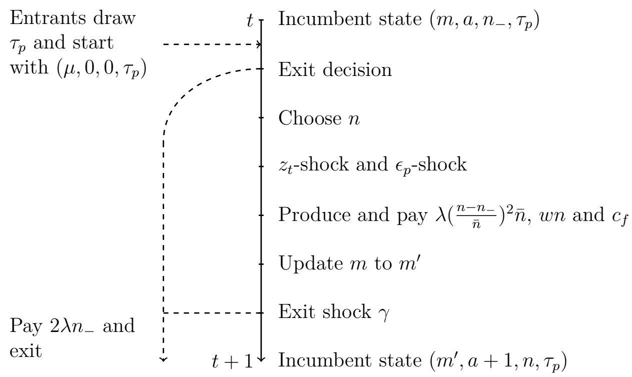


Figure 4: Intra-period timing

the revenue distortion, τ_p. The value of staying, $W(\cdot)$, is given by

$$W(m,a,n_-,\tau_p) = \Bigg[\max_n C^{\frac{1}{\sigma}} P e^{\tau_p} E_z[(e^z)^{\frac{\sigma-1}{\sigma}}] n^{\frac{\sigma-1}{\sigma}} - n - c_f - \lambda(\frac{n-n_-}{\bar{n}})^2 \bar{n} \\ + \beta(1-\gamma) \int V(m', a+1, n, \tau_p) dF(m'|\rho m, \rho \Sigma K) \Bigg]. \quad (25)$$

The solution to this Bellman equation gives an exit policy $x(m, a, n_-, \tau_p)$, taking value 1 if the firm chooses to exit and value 0 if the firm chooses to continue, and an employment policy $n(m, a, n_-, \tau_p)$.

4.3 Entry

There is a continuum of potential entrants. Each of them has to pay entry cost, c_e, if they want to start operating. After paying the entry cost, a new firm starts as an incumbent firm in the next period with age zero and $m = 0$. The entrant learns its permanent revenue wedge, τ_p, only after entry. The amount of entrants is such that the expected gains are equal to the entry cost:

$$c_e = \beta \int V(\mu_{z_p}, 0, 0, \tau_p) H(d\tau_p), \quad (26)$$

where $H(\cdot)$ is the density function for permanent revenue wedge.

4.4 Stationary Equilibrium

Using firms' exit and hiring policies, $x(\cdot)$ and $n(\cdot)$, we can define the evolution of the firm distribution measured at the beginning of each period, $\Psi(dm, a, dn_-, d\tau_p)$. At the stationary equilibrium $\Psi(dm, a, dn_-, d\tau_p)$ is given by

$$\Psi(\mathcal{M}', A', \mathcal{N}, T_p) = \sum_{a|a+1\in A'} \int_{(m,n_-,\tau_p,\tau_t)|n(\cdot)\in\mathcal{N},m'\in\mathcal{M}',\tau_p\in T_p)} [Q(m,\mathcal{M}')(1-x(m,a,n_-,\tau_p))\Psi(dm,a,dn_-,d\tau_p) + M\mathbb{I}(a+1=0)\mathbb{I}(m'=0)\mathbb{I}(n=0)\int_{T_p} H(d\tau_p)], \tag{27}$$

where $Q(m, \mathcal{M}')$ is the transition function for beliefs, each (m, a, n_-, τ_p) is such that $n(m, a, n_-, \tau_p) \in \mathcal{N}$ and $\tau_p \in T_p$. Moreover, M denotes the measure of entrants, $\mathbb{I}(a+1 = 0)$ is an indicator function taking the value of one if $0 \in A'$, $\mathbb{I}(m' = 0)$ is getting the value of one if $0 \in \mathcal{M}'$ and $\mathbb{I}(n = 0)$ takes the value of one if $0 \in \mathcal{N}$.

Given the measure of firms, we can express the labor demand as

$$N = \sum_a \int [n(m,a,n_-,\tau_p) + c_f + \lambda(\frac{n-n_-}{n_-})^2 n_-]\Psi(dm,a,dn_-,d\tau_p) + c_e M. \tag{28}$$

The stationary equilibrium can be defined with policy functions $x(\cdot)$ and $n(\cdot)$, price index, aggregate output, a stationary distribution of firms and a mass of entrants such that

1. The policy rules $x(\cdot)$ and $n(\cdot)$ solve the firm's problem given by (24) and (25)

2. The price level and aggregate output are such that the free entry condition holds

3. The stationary measure of firms is given by (27)

4. the mass of new entrants is such that the labor market clears, i.e., N

given by (28) is equal to fixed labor supply $\bar{N}$

5 Quantitative Analysis

In this section, we match our model with the Finnish firm-level data. We then redo our static calculations using simulated data and explore the importance of different frictions for the determination of the aggregate TFP.

5.1 Calibration Strategy

The parameters governing the preferences of a representative household are calibrated directly, as is the exogenous exit rate. We use the method of simulated moments to fix the rest of the parameters (directly) related to the firms' problem by setting our model to match the prime observable features of the Finnish firm-level data: growth patterns of young and old firms, the size distribution of established firms and the selection patterns. We calculate our targets from the same data set we used in Section 3.

In line with our data, the model's period is set to one year. We assume a 5% real interest rate and thus fix β at 0.95. We follow HK and set the elasticity of substitution, σ, at 3. In addition,as stated earlier, we have normalized the productivity process by setting its unconditional mean to zero.

We use the exit rate of big firms, i.e., firms with a staff of over 50 people, to set the exogenous exit rate, γ. In our data, this exit rate is 3%. After this, we are left with seven parameters to calibrate internally. That is, we still need to determine values for the persistent productivity process, ρ and σ_{ϵ_p}; the variance of temporary productivity, $\sigma^2_{z_t}$; the variance of revenue wedge, $\sigma^2_{\tau_p}$; entry costs, c_e; fixed costs, c_f; and the parameter governing the adjustment costs, λ. To fix the values of these parameters, we minimize the squared relative distance between moments calculated from the Finnish firm-level data and the same moments generated by our model using the identity matrix as a weighting matrix. We target 11 statistics related to the variation of growth rates of young firms (age < 5) and old firms (age ≥ 5), average growth of young firms, the autocorrelation of employment, the entry rate of firms, the difference in exit rates between young and old firms and the size distribution of established firms. Due to a complicated equilibrium setup, the parameter values are defined jointly. However, next, we give a heuristic argument about which statistics are the most relevant for which parameters.

Let us start by considering parameters relating to the productivity process. A noisier signal, higher $\sigma^2_{z_t}$, amplifies uncertainty and the option value of waiting making firms more reluctant to exit early in their life cycle. This reduces the gap between the exit rates of young and old firms. The variance of innovations in the persistent technology component, $\sigma^2_{\epsilon_p}$, increases the weight that firms give to new information, as well as the prevalence of big innovations in the observed process, $z - m$. These effects boost the variance of growth rates for young and old firms. The persistence of the AR(1) process, ρ, also increases the variation of observed innovations, and, thus, the variances of growth rates increase for both age groups. However, unlike $\sigma^2_{\epsilon_p}$, ρ also has a substantial effect on the mean growth rate of new firms and the variance of employment distribution.

The entry rate of new firms is directly related to the entry costs. In addition, increasing entry costs reduces competition and boosts prices. This increases the optimal size of all firms, which in turn fosters the growth rate of young firms. The shape of the employment distribution for old firms is sensitive to fixed costs. Increasing fixed costs increases endogenous exits and tilts the firm distribution towards large firms. A higher variation in revenue distortions increases the first two moments of the employment distribution. Finally, adjustment costs mainly affect the autocovariance of employment.

Before moving on to the fit of the model, it is worth highlighting that our targets do not include ex post misallocation nor its components. Our goal is to see whether a learning model that is set to reproduce the basic life-cycle facts of firms' growth is naturally able to generate uncertainty and misallocation patterns in line with our accounting exercise.

5.2 Fit of the Model

Table 4 gives the data targets and the model counterparts. The associated parameter values are given in Table 5. Overall, the model fits the data quite well, especially taking into account the over-identification. To be more precise, our model generates a size distribution that is similar to the Finnish data. The model also captures the targeted growth patterns; the mean growth rates for young firms and the average variation in growth rates for young and old firms are closely matched. The same is true for the autocorrelation. Regarding selection, our model is able to reproduce the difference in exit rates between young and old firms. The only target for which the model's fit is somewhat weaker is the entry rate, which is lower in the model than in the data.

In line with the different growth patterns for young and old firms present in the data and in the model, our calibration implies a substantial learning over the life-cycle. One way to summarize this is to look at the ratio $\frac{\Sigma_\infty+\sigma_{z_t}}{\Sigma_0+\sigma_{z_t}}$, i.e., the variance of forecast error in productivity for old firms with converged uncertainty relative to the entrant's variance. With our parametrization, this ratio is 43%.

5.3 Static Misallocation in the Simulated Data

We now explore the misallocation patterns in simulated data generated by our model and compare these with the ones we observed in Finnish data. In this regard, we redo our calculations of static misallocation that allowed us to decompose ex post misallocation to ex ante misallocation and uncertainty. We use the benchmark empirical results with the pooled data given in Section 3.2 against which we compare our model.

Table 6 reports the indirect measure of ex post misallocation and its decomposition in data generated with our calibrated model. The variance of the HK revenue wedge is 0.184. In the Finnish data with our baseline setup, it was 0.207 when looking at all industries and years jointly. Thus, our model is able to explain around 90% of the observed variation in the HK revenue wedge.

Table 4: Targets and model counterparts

Moment	Data	Model
Firm Growth		
Mean growth rate, young firms	0.09	0.10
Std of growth rates, young firms	0.35	0.35
Std of growth rates, old firms	0.25	0.22
Autocorrelation of employment	0.96	0.97
Size distribution, old firms		
Firms with employment < 5	0.57	0.57
Firms with employment ≥ 5 and < 10	0.20	0.18
Firms with employment ≥ 10 and < 20	0.12	0.15
Firms with employment ≥ 20 and < 50	0.07	0.08
Firms with employment > 50	0.04	0.03
Selection		
Diff. of exit rates between young and old	0.02	0.02
Entry rate	0.07	0.04

Notes: Firms are called young (old) when they are under (over) 5 years old.

Table 5: Parameter values

Description	Parameter	Value
Entry cost	c_e	11.45
Fixed cost	c_f	0.90
Std. of innovations to AR(1)	σ_{ϵ_p}	0.17
Persistence of AR(1)	ρ	0.92
Signal noise	σ_{z_t}	0.36
Tax-like wedges	σ_{τ_p}	0.55
Adjustment costs	λ	0.08

Looking at the components of ex post misallocation, one observes that, without targeting the decomposition, the model generates levels of uncertainty and ex post misallocation that are really close to the ones observed in the pooled data. The model implied the variance of $1 - \varphi$ is 0.093. In the pooled

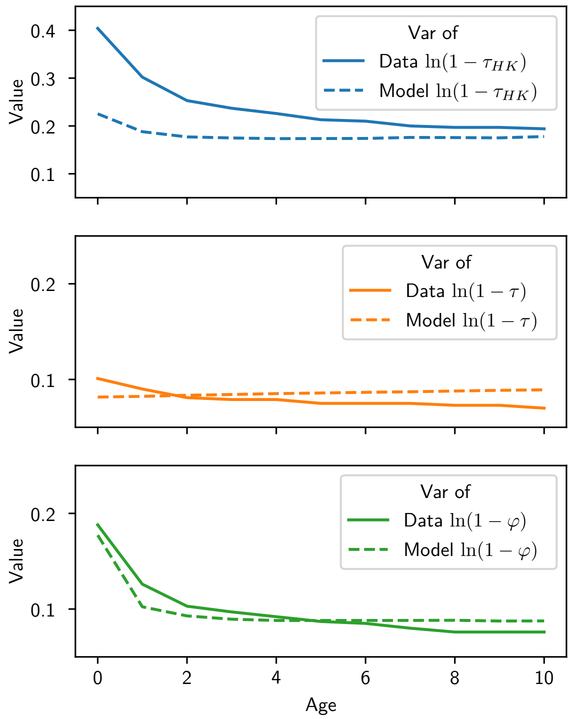


Figure 5: Uncertainty and misallocation conditional on age: model vs data.

data, this was 0.084. The variance of the tax-like wedge is also 0.093, while it was 0.075 in Section 3.2. The main difference between the model and the empirical results is in the covariance term, which was 0.045 , while here it is essentially zero. Due to this the relative importance of uncertainty and ex ante misallocation is slightly higher in the model when compared to the empirical equivalent.

In Figure 5, we replicate the exercise shown in Figure 1. We calculate the variance of the HK revenue wedge conditional on the age of the firms and decompose it to a component reflecting uncertainty and the dispersion in the residual wedge. To facilitate comparisons with the empirical decomposition, we also reproduce the results given in Figure 1.

Table 6: The variance of the HK style revenue wedge and its decomposition to uncertainty and residual wedge calculated from the simulated data

Variable	Value	Share of Var($\ln(1-\tau_{HK})$)
Var($\ln(1-\tau_{HK})$)	0.184	1.00
Var($\ln(1-\tau)$)	0.093	0.51
Var($\ln(1-\varphi)$)	0.093	0.51
2Cov($\ln(1-\tau), \ln(1-\varphi)$)	-0.001	-0.01

Starting with uncertainty, Figure 5 reveals that our model generates an age-dependent pattern that is similar to the one in the data. For example, the variance of $1-\varphi$ for new firms is 0.18 in the simulated data, while in Section 3.2, it was 0.19. The largest deviation between the model and the data is in the second year in which the model produces 77% of the observed variation in $1-\varphi$ (0.10 vs 0.13). In line with the aggregate results in Table 6 and the fact that there is no strong age-dependent trend in the observed ex ante misallocation, the life-cycle profile of ex ante wedge produced by the model is also close to the empirical one.

Finally, we see that the clearest quantitative difference between the model and the data is in ex post misallocation in the early life-cycle. For example, in the first year, the model generates about 58% of the observed ex post misallocation (0.23 vs 0.40). Given the model's ability to replicate the two independent components of ex post misallocation, uncertainty and ex ante misallocation, the difference between the empirical accounting exercise and the one implied by the model rises from the covariance term. As Section 3.3 already highlighted, one obvious explanation for this could be an unmodeled heterogeneity, for example, in financial costs or production technology. Moreover, measurement errors in employment could also play a role. For instance, for startups, the owners' own labor effort is usually a highly important component of aggregate annual labor. However, this labor effort is typically not included in our measure of labor input.[19]

5.4 The Quantitative Significance of Uncertainty, Misallocation and Adjustment Costs

In order to evaluate how different factors affect productivity, we set up a benchmark economy without distortions or uncertainty, where in each period, firms first observe their productivity components and then choose their labor input. Thus, there is no learning nor intra-period uncertainty. Moreover, there are no adjustment costs or tax-like revenue distortions. Otherwise, we use our calibrated parameter values. We normalize the aggregate TFP of this economy to 100 and then illustrate how adding frictions affects the TFP in relative terms. Since the labor supply is fixed, this also gives the output responses. Table 7 assembles the results of these counterfactuals. In all cases, we use the parameter values given in Table 5 to fix the levels of different frictions.

From Table 7, we see that when there are no other frictions, uncertainty reduces the aggregate productivity by 12%. Interestingly, this is close to the cost implied by the back-of-the-envelope calculation discussed in Section 3.2. The table also highlights the importance of the interaction between different wedges in a dynamic setting. If ex ante misallocation and adjustment costs are already present, the introduction of uncertainty only reduces the aggregate TFP by 8%.

Even though in the model, as well as in the data, uncertainty and ex ante misallocation were equally important for ex post misallocation, their impact on aggregate TFP is quite different. Compared to the benchmark, the introduction of tax-like distortions leads to a 41% reduction in productivity. It is also worth noting that this effect does not depend on the presence of other wedges. Adding ex ante misallocation to an economy where we already have uncertainty and adjustment costs still reduces productivity by 40%.

[19]The measurement error in labor effort also rises due to heterogeneity in human capital. In the case of a competitive labor market, this can be controlled by using wage stock as a measure of labor input. However, with frictional labor markets this connection is more complicated.

The main reason why uncertainty is substantially less costly compared to ex ante misallocation in the model is the fact that uncertainty distorts resource allocation only temporarily. In our calibration, firms learn their current persistent productivity relatively fast and adjust their size accordingly. When the learning happens more slowly, the costs associated with uncertainty increase substantially.

Finally, adjustment costs alone drop aggregate productivity by 5%. The relative drop is of the same magnitude if adjustment costs are added to an economy where we already have ex ante misallocation in place. However, adding adjustment costs to a setup where we have misallocation together with uncertainty does not lead notable reduction in TFP.

Table 7: Results: the frictions and aggregate TFP

	No uncertainty	Uncertainty
No distortions or adjustment costs	100.00	87.79
Adjustment cost	95.01	86.14
Misallocation	58.72	51.55
Misallocation and adjustment costs	55.71	51.36

Notes: Uncertainty refers to a case where firms do not know their persistent productivity and choose their labor before they know the current period productivity. In order to shut down tax-like distortions, we set $\sigma_{\tau_p} = 0$.

6 Conclusions

In this paper, we develop a method to quantify the level of idiosyncratic uncertainty and distinguish it from firm-level ex ante wedges. The approach builds on a minimum amount of theory, just specifying demand structure and production function. In our setup, the two key empirical ratios to pin down uncertainty and ex ante misallocation are profits-to-wage-stock and value-added-to-wage-stock. We then explore the importance of uncertainty and ex ante misallocation for the observed ex post misallocation utilizing Finnish administrative data that gives us annual high-quality information of nearly all Finnish firms. According to our findings, uncertainty plays as important role as ex ante misallocation, accounting for about 40% of the ex post misallocation. Moreover, we also show that there is a strong decreasing age-dependent trend in uncertainty. These observations are robust for several alternative specifications.

To understand these empirical results, we set up a life-cycle model of firm dynamics, where firms are uncertain about their fundamentals. We match our model with the growth profiles, the size distribution of mature firms and the selection in the Finnish data and show that the model produces patterns of uncertainty and ex ante misallocation in line with our empirical observations. We also use the calibrated model to evaluate the importance of uncertainty and misallocation for aggregate TFP. According to our counterfactuals, ex ante misallocation reduces TFP by 41%, while the contribution of uncertainty varies between 8-12%.

Even though counterfactuals with our calibrated model suggest that uncertainty is substantially less costly than ex ante misallocation, output losses from uncertainty are still significant. It seems plausible that idiosyncratic uncertainty, at large, is a part of the economic environment and is not mainly directly driven by policies. However, this does not mean that some policies might not indirectly affect uncertainty. For example, any policies that affect the age distribution of firms also influence the observed variation in prediction errors, as older firms generally make more precise forecasts. Thus, startup grants and other government support for new businesses also indirectly contribute to misallocation through their effect on the age distribution.

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Appendix A. Additional Descriptives

This appendix provides basic descriptive statistics for the key variables and the ratios we use to pin down our measures of ex ante misallocation and uncertainty. These statistics are assembled in Table 8.

Yearly Average Number of Firms:		
40156		
Descriptives for Central Variables:		
in millions, no winsorization		
Variable	Mean	Variance
py	1.14	430.92
π	0.16	174.71
wl	0.72	68.49
k	3.95	14756.62
$p^m m$	2.55	5855.52
$(py)_{\text{sales}}$	5.03	13071.06
Descriptives of Key ratios:		
1% winsorization		
Mean:	$\frac{\pi}{wl}$	$\frac{py}{wl}$
	0.222	1.525
Var:	$\frac{\pi}{wl}$	$\frac{py}{wl}$
	0.343	0.812
Cov:	$\frac{\pi}{wl}, \frac{py}{wl}$	
	0.443	

Table 8: Descriptives

After filtering the data, we have around 320 000 firm-year observations and, on average, 40 000 firms per year. The average firm has 18.8 employees and is 7.14 years old. The mean of value added generated by these firms is 1.14 million euros, which is substantially less than the mean value of sales (5 million). This highlights the role of materials in production (the mean is 2.5 million). For all of these variables, the variation around their means is substantial. The same is true for capital, which we measure using total assets.

Mean employment costs are 0.7 million euros, and mean profits are 1.14 million euros. There is, again, substantial variation in annual profits across firms.

The lower part of the table provides the means and variances for py/wn and π/wn, as well as the covariance between these two variables. In line with our main results, we have winsorized these variables. The mean profit-to-wage ratio is substantially smaller than the mean value-added-to-wage-bill ratio. There is also notably more variation in the value-added-to-wage-bill ratio than in the profit-to-wage ratio. As illustrated in the main text, variation in the former ratio informs us about ex post misallocation, while variation in the latter ratio provides us with insights into uncertainty.

Appendix B. Misallocation for Different Industries

This appendix explores the static misallocation and its decomposition for different industries. In addition, we also compare exporting and non-exporting firms.

Table 9 reproduces the static decomposition of Table 1, but focuses on one industry at a time. The main observation from the table is that, excluding electricity, gas and water supply, the results are remarkably stable across industries. Ex post misallocation levels are close to the level of pooled data (though slightly higher in wholesale and retail trade). The same is also true for the components of ex post misallocation. Overall, it seems that uncertainty and ex ante misallocation are more or less equally important, both accounting for around 40% of the total variation. The relative importance of covariance term is typically around 20%.

Given these general patterns, electricity, gas and water supply (Industry E) stands out. The variance of $\ln(1-\tau)$ is more than 4 times higher than in other industries. Consistent with this, ex post misallocation is also at a substantially higher level. A distinctive feature of this industry is that it is much more heavily regulated than other industries. In addition, competition

Table 9: Ex post misallocation and its decomposition for different industries.

Industry	$\text{Var}(\ln(1-\tau_{HK}))$	$\text{Var}(\ln(1-\tau))$	$\text{Var}(\ln(1-\varphi))$	$2\text{Cov}(\ln(1-\tau),\ln(1-\varphi))$
D Manufacturing	0.179	0.072 [40%]	0.073 [41%]	0.033 [18%]
E Electricity, gas and water supply	0.431	0.342 [79%]	0.080 [19%]	-0.005 [-1%]
F Construction	0.175	0.059 [34%]	0.073 [42%]	0.039 [22%]
G Wholesale and retail trade	0.269	0.090 [33%]	0.103 [38%]	0.072 [27%]
H Hotels and restaurants	0.209	0.072 [34%]	0.086 [41%]	0.047 [23%]
I Transport, storage and communication	0.184	0.080 [44%]	0.069 [38%]	0.029 [16%]

Notes: The relative contributions are given in brackets. Percentages do not necessarily sum up to one because of the winsorization.

is limited at best: gas supply is in the hands of a nationwide monopoly, while water supply and electric grid are run by local monopolies. Moreover, local governments own the water supply companies and are also often major owners of electricity companies. These sector-specific features are likely behind the elevated level of misallocation. Another difference between Industry E and the rest of the economy is the negative covariance term. Uncertainty, on the other hand, is on a par with other industries. Together these facts imply that mots of the variation, almost 80%, in the HK wedge is accounted for by ex ante misallocation.

One of the characteristic features of the Finnish economy that sets it apart from large countries such as the US, China or India is the relatively high importance of exports. This might be relevant for us if wedges (ex ante wedges or prediction errors) or their joint variation differ systemically between exporters and non-exporters. For example, markups could differ between exporters or non-exporters (see De Loecker and Warzynski, 2012) or currency fluctuations might affect prediction errors differently. Our data allows us to observe firms' exports for years 1995-2007. However, when calculating our decomposition for exporters alone, we observe that this dimension does not affect our results. For pooled data over years 1995-2007 with exporters only, we get $\text{Var}(\ln(1-\tau_{HK})) = 0.216$, $\text{Var}(\ln(1-\tau)) = 0.085$, $\text{Var}(\ln(1-\varphi)) = 0.085$ and $2\text{Cov}(\ln(1-\tau), \ln(1-\varphi)) = 0.043$.

Appendix C. Year and Cohort Effects in Misallocation

This appendix reports the results of the baseline accounting exercise for selected years and cohorts.

Table 10: The variance of HK style revenue wedge and its decomposition to uncertainty and residual wedge for different years.

	$\text{Var}(\ln(1-\tau_{HK}))$	$\text{Var}(\ln(1-\tau))$	$\text{Var}(\ln(1-\varphi))$	$2\text{Cov}(\ln(1-\tau), \ln(1-\varphi))$
1996	0.309	0.074 [24%]	0.157 [51%]	0.072 [23%]
2001	0.190	0.070 [37%]	0.073 [38%]	0.042 [22 %]
2006	0.198	0.077 [12%]	0.078 [55%]	0.040 [23%]
2011	0.194	0.079 [41%]	0.074 [38%]	0.038 [20%]

Notes: Percentages do not necessarily sum up to one because of the winsorization. Firm-specific α refers to firm-time-specific capital elasticities.

In Table 10 we measure misallocation in different years. The dispersion of the residual wedge, our measure of ex ante misallocation, is again stable over the subsets of the data. Moreover, the values are close to the ones observed in the pooled data. The same is true for the measure of uncertainty and the covariance term with the exception of the year 1996. In 1996, Finland was still recovering from the most severe recession of its peacetime history and the structural change triggered by the collapse of trade with the Soviet Union was still underway (see, e.g., Gorodnichenko et al, 2012). In light of this, an increased level of uncertainty is perhaps to be expected. [20]

Figure 6 presents the evolution of the decomposition for selected cohorts of firms. We have chosen these cohorts such that the starting years are in line with the years reported in Table 10. The figure illustrates that our main conclusion, decreasing trends in ex post misallocation and uncertainty, is present for all cohorts. It is worth noting, though, that the first year of the 1996 cohort stands out. From Table 10, we already know that this year ex post misallocation and uncertainty were exceptionally high. This is clearly true for the startups as well. Interestingly, however, at their second year and thereafter, the 1996 cohort does not stand out when compared against other cohorts.

Appendix D. Labor distortions

Like most of the literature, we have not allowed for the use of labor to be affected by an output wedge and an input wedge jointly. Instead, we have focused on the decomposition between the prediction error and the output wedge. In this appendix, we generalize our accounting exercise to also include labor distortions.

With this extension, the HK wedge identified from the first order condition of labor is a combination of output distortion $(1 - \tau^{y}_{t,s,i})$, labor distortion $(1 + \tau^{n}_{t,s,i})$ and prediction error $(1 - \varphi_{t,s,i})$. To separate these wedges, we consider a more general production function, where a firm also uses materials, m, in addition to labor, n, and capital, k. We assume that a firm chooses its materials after uncertainty has been revealed. Moreover, we also assume that the firm's use of materials is not distorted by an input wedge. Now we can identify the output distortion from the first order condition of materials. Next, the ex post profit condition allows us to pin down the prediction error. When the prediction error and the output wedge are uncorrelated, violation of our identifying assumptions would underestimate uncertainty and overestimate

[20] We also observe similar high values for year 1995. From 1997 onwards the observed values are in line with 2006 and 2011.

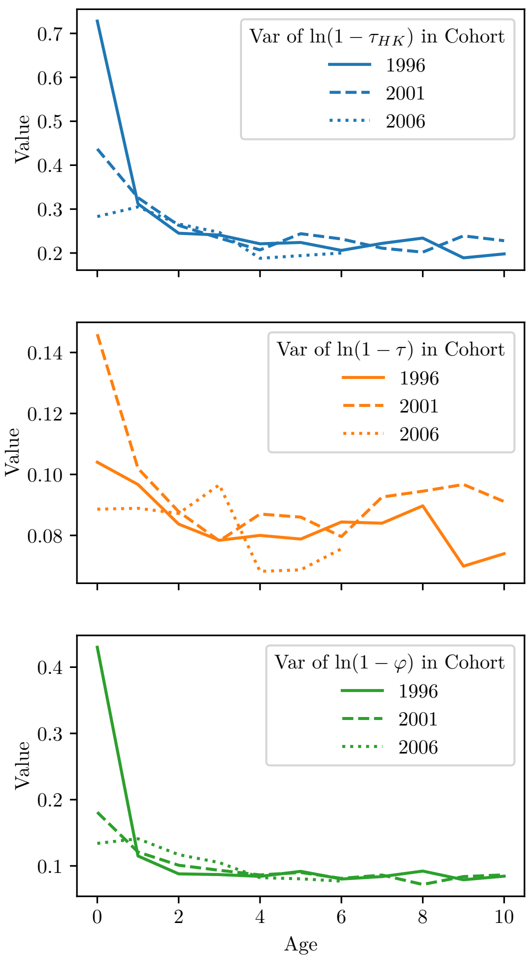

Figure 6: Life-cycle of uncertainty and misallocation for selected cohorts.

misallocation arising from output wedges. Finally, the first order condition with respect to labor enable us to quantify the labor wedge.

The firm problem is now given by

$$\max_{n,k} \left\{ E\left[\max_{m}(1-\tau^{y}_{t,s,i})p_{t,s,i}y_{t,s,i} - p^{m}_{t}m_{t,s,i}\right] - (1+\tau^{n}_{t,s,i})w_{t,s,i}n_{t,s,i} - R_{t}K_{t,s,i}\right\}$$

As in the main text, firms face CES demand, thus $p_{t,s,i} = \frac{y_{t,s,i}}{Y_t}^{-\frac{1}{\sigma}}P_t$. The production technology is given by a Cobb-Douglas production function, $y_{t,s,i} = z_{t,s,i}k^{\alpha_s}_{t,s,i}n^{\xi_s-\alpha_s}_{t,s,i}m^{1-\xi_s}_{t,s,i}$

At the second stage, the first order condition with respect to intermediate inputs can be written as

$$(1-\tau^{y}_{t,s,i})(1-\frac{1}{\sigma})(1-\xi_s)P_tY_t^{\frac{1}{\sigma}}z^{1-\frac{1}{\sigma}}_{t,s,i}k^{\alpha_1}_{t,s,i}n^{\alpha_2}_{t,s,i}m^{\alpha_3}_{t,s,i} = p^{m}_{t}m_{t,s,i}, \tag{29}$$

where $\alpha_1 \equiv \alpha_s(1-\frac{1}{\sigma_{t,s,i}})$, $\alpha_2 \equiv (\xi_s-\alpha_s)(1-\frac{1}{\sigma_{t,s,i}})$ and $\alpha_3 \equiv (1-\xi_s)(1-\frac{1}{\sigma_{t,s,i}})$. From the previous equation we can solve the output wedge

$$(1-\tau^{y}_{t,s,i}) = \frac{\sigma}{\sigma-1}\frac{1}{1-\xi_s}\frac{1}{\frac{p_{t,s,i}y_{t,s,i}}{p^{m}_{t}m_{t,s,i}}}. \tag{30}$$

Thus, for a given parametrization, the ratio of revenues to material costs gives us a measure of the output wedge. In the main text, this wedge was pinned down by the ratio of value added to wage stock.

Moreover, from the first order condition we get

$$m_{t,s,i} = \left[\frac{\alpha_3}{p^{m}_{t}}(1-\tau^{y}_{t,s,i})z_{t,s,i}P_tY_t^{\frac{1}{\sigma}}z^{1-\frac{1}{\sigma}}_{t,s,i}k^{\alpha_1}_{t,s,i}n^{\alpha_2}_{t,s,i}\right]^{\frac{1}{1-\alpha_3}}$$

Using eq (29), the first stage problem can be written as

$$
\begin{aligned}
&\max_{n,k}[(1-\tau^y_{t,s,i})Ep_{t,s,i}y_{t,s,i} - \alpha_3 Ep_{t,s,i}y_{t,s,i}] - (1+\tau^n_{t,s,i})w_{t,s,i}n_{t,s,i} - R_t k_{t,s,i} \\
=&\max_{n,k}(1-\tau^y_{t,s,i})(1-\alpha_3)P_t Y_t^{\frac{1}{\sigma}} E\left\{ z_{t,s,i}^{1-\frac{1}{\sigma}} k_{t,s,i}^{\alpha_1} n_{t,s,i}^{\alpha_2} \left[\frac{\alpha_3}{p^m_t} z_{t,s,i}^{1-\frac{1}{\sigma}} k_{t,s,i}^{\alpha_1} n_{t,s,i}^{\alpha_2}\right]^{\frac{\alpha_3}{1-\alpha_3}} \right\} \\
&\quad - (1+\tau^n_{t,s,i})w_{t,s,i}n_{t,s,i} - R_t k_{t,s,i} \\
=&\max_{n,k}(1-\tau^y_{t,s,i})^{\frac{1}{1-\alpha_3}}(1-\alpha_3)P_t^{\frac{1}{1-\alpha_3}} Y_t^{\frac{1}{\sigma}\frac{1}{1-\alpha_3}} E z_{t,s,i}^{\frac{1}{1-\alpha_3}(1-\frac{1}{\sigma})} (\frac{\alpha_3}{p^m_t})^{\frac{\alpha_3}{1-\alpha_3}} k_{t,s,i}^{\frac{\alpha_1}{1-\alpha_3}} n_{t,s,i}^{\frac{\alpha_2}{1-\alpha_3}} \\
&\quad - (1+\tau^n_{t,s,i})w_{t,s,i}n_{t,s,i} - R_t k_{t,s,i}.
\end{aligned}
$$

For $\tilde{a}_i \equiv \frac{\alpha_i}{1-\alpha_3}$, the first order condition with respect to labor is given by

$$
\begin{aligned}
\tilde{\alpha_2}(1-\tau^y_{t,s,i})^{\frac{1}{1-\alpha_3}}(1-\alpha_3)P^{\frac{1}{1-\alpha_3}} Y^{\frac{1}{\sigma}\frac{1}{1-\alpha_3}} E z_{t,s,i}^{\frac{1-\frac{1}{\sigma}}{1-\alpha_3}} (\frac{\alpha_3}{p^m_t})^{\tilde{\alpha_3}} k_{t,s,i}^{\tilde{\alpha_1}} n_{t,s,i}^{\tilde{\alpha_2}} =& (1+\tau^n_{t,s,i})w_{t,s,i}n_{t,s,i} \\
(1-\alpha_3)\tilde{\alpha_2}(1-\tau^y_{t,s,i})PY^{\frac{1}{\sigma}} \frac{E z_{t,s,i}^{\frac{1-\frac{1}{\sigma}}{1-\alpha_3}}}{z_{t,s,i}^{\frac{1-\frac{1}{\sigma}}{1-\alpha_3}}} z_{t,s,i}^{1-\frac{1}{\sigma}} k_{t,s,i}^{\alpha_1} n_{t,s,i}^{\alpha_2} m_{t,s,i}^{\alpha_3} =& (1+\tau^n_{t,s,i})w_{t,s,i}n_{t,s,i} \\
(1+\tau^n_{t,s,i}) = \alpha_2(1-\tau^y_{t,s,i})(1-\varphi_{t,s,i})\frac{p_{t,s,i}y_{t,s,i}}{w_{t,s,i}n_{t,s,i}}&. \qquad (31)
\end{aligned}
$$

Rearranging this equation illustrates that the HK wedge, can now be written as

$$
1-\tau^{HK}_{t,s,i} \equiv \frac{\sigma w_{t,s,i}n_{t,s,i}}{(\sigma-1)(\xi_s-\alpha_s)p_{t,s,i}y_{t,s,i}} = \frac{(1-\tau^y_{t,s,i})(1-\varphi_{t,s,i})}{(1+\tau^n_{t,s,i})}.
$$

The first order condition with respect to capital is given by

$$
\begin{aligned}
\tilde{\alpha_1}(1-\tau^y_{t,s,i})^{\frac{1}{1-\alpha_3}}(1-\alpha_3)P^{\frac{1}{1-\alpha_3}} Y^{\frac{1}{\sigma}\frac{1}{1-\alpha_3}} E z_{t,s,i}^{\frac{1-\frac{1}{\sigma}}{1-\alpha_3}} (\frac{\alpha_3}{p^m_t})^{\tilde{\alpha_3}} k_{t,s,i}^{\tilde{\alpha_1}} n_{t,s,i}^{\tilde{\alpha_2}} =& R_t k_{t,s,i} \\
\alpha_1(1-\tau^y_{t,s,i})(1-\varphi_{t,s,i})p_{t,s,i}y_{t,s,i} =& R_t k_{t,s,i}. \qquad (32)
\end{aligned}
$$

As in the main text, we can write realized profits with the help of optimal policies:

$$\begin{aligned}\pi_{t,s,i} =&(1-\tau^y_{t,s,i})p_{t,s,i}y_{t,s,i}\left[1-\alpha_3-(\alpha_1+\alpha_2)(1-\varphi_{t,s,i})\right]\\ \pi_{t,s,i} =&\frac{1}{\alpha_3}p^m_t m_{t,s,i}\left[1-\alpha_3-(\alpha_1+\alpha_2)(1-\varphi_{t,s,i})\right],\end{aligned}\tag{33}$$

where in the second equation we have used eq (31). Solving $(1-\varphi_{t,s,i})$ from this equation gives

$$(1-\varphi_{t,s,i}) = \frac{\frac{\sigma}{\sigma-1}-(1-\xi_s)(1+\frac{\pi_{t,s,i}}{p^m_t m_{t,s,i}})}{\xi_s}.\tag{34}$$

In our extended framework, the ratio of profits to material costs gives us a measure of prediction error. This is different from our baseline accounting exercise, where the key ratio was between profits and wage stock.

Finally, with measures of the output wedge and the prediction error, we can solve the labor wedge from the first order condition regarding labor, eq (31),

$$\begin{aligned}(1+\tau^n_{t,s,i}) =&\alpha_2(1-\tau^y_{t,s,i})(1-\varphi_{t,s,i})\frac{p_{t,s,i}y_{t,s,i}}{w_{t,s,i}n_{t,s,i}}\\ (1+\tau^n_{t,s,i}) =&\frac{\xi_s-\alpha_s}{1-\xi_s}\frac{\frac{\sigma}{\sigma-1}-(1-\xi_s)(1+\frac{\pi_{t,s,i}}{p^m_t m_{t,s,i}})}{\xi_s}\frac{p^m_t m_{t,s,i}}{w_{t,s,i}n_{t,s,i}}\end{aligned}\tag{35}$$

Taking our generalized approach to the data requires us to measure firms' revenues and their use of materials, in addition to profits and wage stocks. Our data set contains these variables for years 1999-2012. In order not to introduce additional variation through parametrization, we utilize the same labor shares used in the main text and scale them to revenue shares with a constant non-material share of 0.3.[21] To keep our results comparable to the main text, we report variation in the HK wedge, $\mathrm{Var}\,(\ln(1-\tau^{HK}))$, and its decomposition to uncertainty,$\mathrm{Var}\,(\ln(1-\varphi))$, the aggregate measure of ex ante misallocation, $\mathrm{Var}\,(\ln(\frac{1-\tau^y}{1+\tau^n}))$, and the covariance between the two components.

[21] That is, we set $\xi = 0.3$ and multiply our original capital and labor elasticities with it.

Table 11 assembles the results of our extended accounting exercise. Using revenues instead of values added increases the variation in the HK wedge notably. The level of uncertainty increases in line with the HK wedge and again makes up for about 40% of the variation in the HK wedge. Next, note that the variation in the combined measure of ex ante wedges is substantial, exceeding the variation in the HK wedge. Given a high union coverage, the importance of centralized wage setting and a rigid labor market regulation, the elevated role of ex ante wedges is perhaps to be expected when labor distortions are also taken into account. Finally, note that now the covariance between the ex ante wedge and the prediction error is negative.

Table 11: Generalized variance decomposition

Variable	Value	Share
$\mathrm{Var}(\ln(1-\tau_{HK}))$	0.467	1.00
$\mathrm{Var}(\ln(\frac{1-\tau_y}{1+\tau_n}))$	0.615	1.32
$\mathrm{Var}(\ln(1-\varphi))$	0.176	0.38
$2\mathrm{Cov}(\ln(\frac{1-\tau_y}{1+\tau_n}), \ln(1-\varphi))$	-0.315	-0.67

Notes: Percentages do not necessarily sum up to one because of the winsorization.

Figure 7 shows the variance terms conditional on age. When revenues are used to measure the variation in the HK wedge, the decreasing trend is more subtle. Uncertainty, though, still decreases substantially.

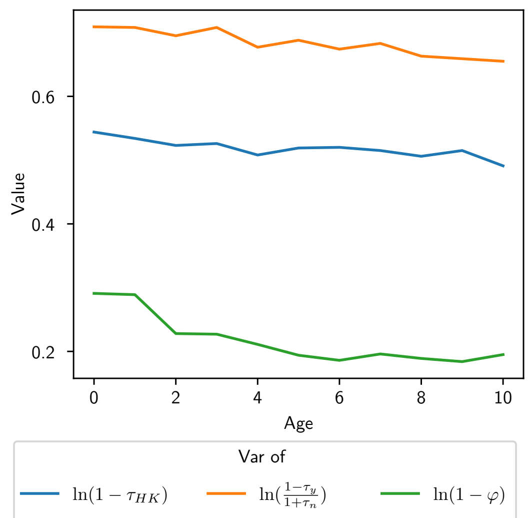

Figure 7: Uncertainty and misallocation conditional on firms' age, generalized setup.