The Price of Knowledge Diffusion: Technology Licensing and Market Power *

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March 25, 2026

Abstract

Business dynamism has been slowing globally over the last several decades. In a recent study, Akcigit and Ates (2023) examine the relative importance of different channels behind this development and highlight weakened knowledge diffusion from the technology frontier to followers as a dominant force.[1] Their study also suggests that diffusion may weaken endogenously as the technology gap widens and market power accumulates, raising the question of how innovation policy can strengthen diffusion without reducing welfare. In this paper we study leader-to-follower licensing as a policy-relevant diffusion margin, and evaluate licensing subsidies relative to direct R&D subsidies. We develop an endogenous-growth general equilibrium model in which firms compete in prices and invest in R&D; the technology leader endogenously chooses whether to license to the follower, trading off higher static profits against faster follower catch-up through knowledge diffusion. We calibrate the model to Finnish data from 2014–2019. Our first exercise evaluates whether allowing licensing is desirable by shutting down the licensing channel in the calibrated economy. In the Finnish benchmark, shutting down licensing lowers growth but increases consumption-equivalent welfare, because the level effects of reduced concentration dominate the diffusion benefits of licensing. We then vary the diffusion rate through licensing and product substitutability to characterize when licensing becomes welfare-improving. In that region, solving the policymaker's problem shows a non-trivial interaction: higher R&D subsidies can reduce equilibrium licensing by moving leaders more quickly into the monopoly-pricing states where licensing is privately unattractive, so the optimal policy mix augments R&D support with a non-negligible licensing subsidy to sustain diffusion.

Keywords: *Antitrust Policy; Business Dynamism; Endogenous Growth; Innovation Policy; Licensing; Technology Diffusion.*
JEL Classification: *E22;L10;L41;O33;O34.*

1 Introduction

The weakening of business dynamism has unfolded gradually over several decades, dating back to at least the 1980s (Akcigit & Ates, 2021). Different factors have been suggested as explanations for this development: reduced effective corporate taxes, large increases in R&D subsidies, diminished labour mobility, and the increasingly strategic use of intellectual property rights, to name a few. In practice, it is likely not a single factor that

*We thank Ufuk Akcigit, seminar participants at Sabanci University, and HMRAD for helpful comments and discussions. We are also grateful to Lieto Savings Bank Foundation for financial support that has made this research possible, and Visa Vilppala for research assistance.

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[1] For evidence on the decline of business dynamism see Decker, Haltiwanger, Jarmin, and Miranda (2016), Philippon (2019), Calvino, Criscuolo, and Menon (2016), and Andrews, Criscuolo, and Gal (2016a).

drives the weakening of business dynamism, but rather several forces operating at the same time. In a recent empirically well-grounded study, Akcigit and Ates (2023) argue that there is a strong common factor behind these channels: weakened knowledge diffusion from the technology frontier to followers. Diminished labour mobility, for example, directly weakens knowledge diffusion as well. They attribute as much as 80% of the decline in most symptoms of business dynamism to this particular factor, regardless of its underlying source.[2] The problem is that we do not really know how to encourage firms to invest in innovation activity while at the same time guaranteeing sufficient diffusion of the results. In the literature this is known as the fundamental dilemma of innovation policy, and there is no simple solution: the goals seem to be in direct conflict with each other, or even contradictory. At a minimum, this suggests that more than one policy tool is needed. Patents, for instance, strongly encourage investment, but at the same time tend to prevent diffusion of results.[3]

To strengthen diffusion, innovation policy must increase the profits that innovators expect to obtain from diffusing their ideas. There is an obvious, yet somewhat neglected, way to achieve this: the government could subsidize licensing.[4] Surprisingly, there does not exist a systematic study comparing the benefits of using public resources to subsidize licensing instead of spending them on direct investment subsidies such as the R&D tax credit. One reason is that there is no consensus on why, and under what circumstances, firms choose to license.[5] Whatever the reason, as a policy tool, subsidies targeted at licensing have several advantages over direct investment subsidies. First, the money is spent only on innovations that have already materialized, rather than on uncertain R&D activity that often leads nowhere. Empirical studies suggest that targeting the right projects is difficult. Kerr, Nanda, and Rhodes-Kropf (2014) estimate that 55% of start-up projects funded by venture capitalists in the US between 1985 and 2009 failed. It is not cheap to spend taxpayers' money like this, especially given how costly it can be for society to raise the funds in the first place.[6] This brings us to a second advantage. Since licensing subsidies increase the payoff from successful innovations, private venture capitalists may invest more in start-ups in response. Moreover, one would expect private venture capitalists to make more reliable estimates about which innovations are likely to succeed than government

[2]For other recent papers that study diffusion see Jo and Kim (2026), Arora, Belenzon, and Sheer (2021) and Baruffaldi and Simeth (2020).

[3]At the time of the Industrial Revolution, when the patent system was established in the UK, there was no consensus among contemporary economists that this was the right approach (Macfie, 1875). More recently Boldrin and Levine (2008, 2013) have argued that there would probably not be much less innovative activity even if patents were not granted at all, a claim that is hard to verify because we never observe this counterfactual world. But this is exactly why theory is needed. For an influential proposal to address the fundamental dilemma within the current patent system, see Kremer (1998) on patent buyouts.

[4]Nagler, Schnitzer, and Watzinger (2022) give anecdotal evidence that knowledge diffusion through licensing can be large.

[5]Palikot and Pietola (2023) study a model where licensing depends on the strength of the patent protecting the innovation. Licensing is used when patent protection is weak in order to avoid litigation (e.g. patent challenges in court). Another reason is that R&D activity often produces innovations that are not directly related to a firm's core business, in which case licensing is natural. The rationale could also be more nuanced than this. Nagler et al. (2022) suggest that licensing can sometimes be used to standardize a technology.

[6]Feldstein (1999) estimates that the deadweight loss of income taxation in the US can be as high as 30% of the revenue collected.

agencies that allocate taxpayers' money.

We often tend to forget that the natural tendency of firms is to acquire market power whenever possible. This means that good antitrust policy plausibly needs an anticipatory element built into it. The weakening of business dynamism is a symptom, an early sign, that we are failing in this respect. Reducing direct R&D support and using the money to subsidize licensing could be interpreted as an anticipatory measure that prevents the productivity gap between leaders and followers from growing too large. The intuition is that subsidizing licensing increases diffusion, while at the same time technology leaders must continue investing in R&D to maintain the leading position if they want to earn licensing income.

In this paper we construct an endogenous-growth general equilibrium model to study how licensing and licensing subsidies affect business dynamism, growth, and welfare. We also compare how licensing subsidies fare relative to direct R&D investment subsidies. We do this by embedding licensing dynamics similar to Arrow (1963) in a model that closely resembles Akcigit and Ates (2023).[7][8] There is a continuum of markets with substitutable products where two incumbent firms (interpreted as best and the rest) that produce a homogeneous good engage in price competition and undertake R&D to improve their production technologies, with the possibility that a new entrant replaces the entire market.[9] The productivity gap between the firms determines pricing power in the form of markups and profits. The follower engages in R&D to catch up to the leader, while the leader engages in R&D to escape competition and increase its markup and profits. This is the basic dynamics in Akcigit and Ates (2023), to which we add the licensing dynamics of Arrow (1963). If the productivity gap between the leader and the follower is large enough, then the monopoly price of the leader is smaller than the marginal cost of the follower. Thus, by setting the monopoly price, the leader can drive the follower out of the market. However, if the productivity gap is not large enough, so that the monopoly price of the leader is larger than the marginal cost of the follower, then the leader cannot charge the monopoly price. In this case, Bertrand competition drives the price down to the marginal cost of the follower. The leader still takes the market, but now it may have an incentive to use licensing. By allowing the follower to extract monopoly rents under a license and extracting revenue through fees, the leader can increase its continuation value. This, however, increases the risk that the follower overtakes the leader because diffusion through licensing can improve the follower's R&D prospects. In our model, the leader solves two dynamic optimization problems at each point in time to compare the continuation value under licensing to the

[7]Similar models have been used recently in Acemoglu and Cao (2015), Klette and Kortum (2004), Lentz and Mortensen (2008), Akcigit and Kerr (2018), and Mukoyama and Osotimehin (2019).

[8]Arrow (1963) studied whether a monopoly or a firm operating in a perfectly competitive market has a stronger incentive to innovate. The result is that a firm operating in a competitive market has a stronger incentive to innovate. The intuition is that a monopolist compares the post-innovation monopoly profit to the pre-innovation monopoly profit, while the comparison point for the competitive firm is the profit it can extract in the competitive environment before market power increases due to the innovation.

[9]This is the creative destruction channel in our model. In Akcigit and Ates (2023) the new entrant replaces only the follower.

continuation value without licensing. Thus, the licensing decision is endogenous.[10]

In our model the leader of each market can use a licensing contract to monetize the market power generated by its productivity advantage. Anecdotal evidence suggests that this is not a far-fetched story. First, antitrust authorities scrutinize how licensing contracts are formulated, suggesting they are at least occasionally used in anti-competitive ways. Second, the form of the most common licensing contract, a fixed fee plus a per-unit royalty, suggests that the contract is designed to reduce quantity by raising marginal costs and shifting surplus toward royalty payments. Therefore, licensing contracts can increase diffusion of knowledge at the expense of higher market concentration. The descriptive statistics that we want our model to replicate point in the same direction (see Fact 3 in Sect. 2).

We use this model to study both the aggregate desirability of allowing licensing and the implications of redirecting innovation support toward licensing subsidies. As a first exercise, we calibrate the model using Finnish data from 2014–2019, set subsidies to their observed levels, and then shut down the licensing channel entirely. Somewhat surprisingly, growth goes down from 0.863% to 0.833%, while welfare measured in consumption-equivalent (CE) units goes up by 0.362%. This demonstrates the danger of focusing too much on growth alone. The result is driven by level effects from concentration: shutting down the licensing channel reduces profits and increases both output and wages (Table 5, Panel A). Although licensing improves diffusion, and therefore improves measures of business dynamism,[11] the adverse effect of higher market concentration dominates the welfare change. In summary, since licensing is welfare-reducing in the benchmark calibration, subsidizing it is not warranted in Finland under the benchmark environment.

This being the case, we ask whether the licensing channel can ever be beneficial in the aggregate. We do this by setting the diffusion channel to a theoretical maximum while leaving all other parameters at their benchmark levels, and then shutting down the licensing channel again. In this high-diffusion environment, licensing is beneficial both in terms of growth and welfare: shutting down licensing reduces growth from 0.931% to 0.833%, and welfare falls 0.455%. Here, the diffusion benefits of licensing dominate the adverse effect of higher concentration (Table 6, Panels A–C).

Next we study how strong diffusion through licensing must be before licensing becomes beneficial. We do this around the benchmark calibration, varying only the parameters that govern diffusion of knowledge through licensing and product substitutability (Figure 1). The result indicates that the welfare effect of licensing depends crucially on product substitutability.[12] It also shows that diffusion through licensing must be substantially stronger for licensing to be beneficial in Finland, almost three times stronger. Although our analysis is economy-wide, the result suggests that licensing could plausibly be beneficial in high-diffusion sectors.

[10] To calibrate the model we match the fraction of licensing firms to what we observe in the data.

[11] Leapfrog rate, for example, jumps from 2.61 to 3.06 (Table 5, Panel B).

[12] Syverson (2004) shows that, at the industry level, productivity dispersion and product substitutability are negatively correlated. Our model replicates this empirical fact.

Then, as the final quantitative exercise, we study whether it is desirable to subsidize licensing rather than use the same resources as a direct R&D subsidy in the region where licensing is beneficial. We again set the diffusion channel to a theoretical maximum while leaving all other parameters at their benchmark levels. We then solve for welfare-maximizing innovation policy in two regimes: one in which licensing-targeted subsidies are allowed and one in which they are not. When licensing subsidies are allowed, the optimal policy assigns a non-negligible subsidy to licensing and welfare measured in CE rises substantially, from 0.570% to 0.808% relative to the benchmark in that high-diffusion environment (Table 7).

Related literature. In a broad sense our paper is related to a growing literature that studies technology gaps and different channels of knowledge spillovers (Acemoglu and Cao (2015), Akcigit and Ates (2023), Andrews, Criscuolo, and Gal (2016b), Arora et al. (2021), Baruffaldi and Simeth (2020), Bessen, Denk, Kim, and Righi (2020), De Ridder (2024), Hegde, Herkenhoff, and Zhu (2023), Jo and Kim (2026)). Although we focus on market concentration rather than competition, our paper also contributes to the extensive literature on the connection between innovation and competition (Acemoglu and Akcigit (2004), Aghion and Howitt (1992), Aghion, Antonin, and Bunel (2023)) and between innovation and markups (Peters (2020)). Likewise, there is an expanding literature on patents and other IPR protection in endogenous-growth general equilibrium models (see Suzuki and Kishimoto (2025), Hegde et al. (2023), Baruffaldi and Simeth (2020), and O'Donoghue and Zweimüller (2004) and references therein), but this literature rarely touches licensing, and it is even rarer for licensing to be endogenous.

Two papers that come closest to what we do here are Acemoglu and Akcigit (2012) and Aghion, Bergeaud, Boppart, Klenow, and Li (2025). Although the main goal of Acemoglu and Akcigit (2012) is to show that IPR protection should be contingent on technological lead due to the *trickle-down effect* (technologically more advanced firms should be granted stronger protection because this also makes less advanced firms invest more to acquire better protection), they also study licensing. The difference is that licensing is compulsory in their model, i.e. the follower makes the licensing decision. In our model it is the leader who makes the licensing decision after evaluating whether it is worth the risk. State-dependent IPR is an instance of targeting policy. There is also a natural targeting policy for licensing: make licensing compulsory for firms that use public funding in R&D. It is not clear whether this policy is desirable, because firms may then refuse public funding and thus innovate less; our model could be used to study this question. Here, however, we want to compare direct R&D subsidies and licensing subsidies when no good targeting strategies exist for either instrument. Direct R&D support can be wasteful because many subsidized projects fail, while licensing support, while going only to those firms that have actually made an innovation (exact ex post targeting), is wasteful because it is also paid to innovations that would have been licensed anyway. We want to learn which channel dominates. The results of Aghion et al. (2025), on the other hand, complement ours nicely. They study a model where markups are generated by two factors, innovation quality and process efficiency, while in our model markups are generated by process efficiency alone. Although they are mainly interested in other questions, they show that licensing can be beneficial when high innovation step-size firms can freely license to high process-efficiency firms.

The rest of the paper is structured as follows. In Section 2 we present descriptive facts from Finnish administrative licensing data that we use for calibration and target setting in the model. In Section 3 we explain how Arrowian dynamics can be embedded in the model of Akcigit and Ates (2023) to study the effects of licensing on innovation intensity, growth, and welfare. In Section 4 we develop and solve the endogenous licensing model. In particular, we define competitive general equilibrium and explain how the licensing fee is determined through Nash bargaining over continuation values. In Section 5 we study aggregates. We develop a decomposition for the growth rate into three components: growth from leader innovation, growth from follower innovations that build on the leader's technology (diffusion through licensing), and growth from follower innovations on their own technology (leapfrogging). We also develop an index to measure aggregate concentration. This is important because the adverse effects of licensing policies operate through this margin. In Section 6 we present the calibration and our main quantitative results. Section 7 concludes.

2 Descriptive Facts on Licensing

We begin by documenting stylized facts on licensing using novel Finnish administrative data that link financial statements and balance sheets to survey and tax records for the universe of firms (details in Appendix A). Throughout, "intensity" denotes a variable divided by revenue (gross output). These are the empirical facts that we want our model to replicate:

Fact 1. Licensing is rare but economically meaningful. A minority of firms license, yet intensities are non-trivial.

Fact 2. Licensing covaries positively with R&D. Firms with higher R&D intensity earn more from licensing.

Fact 3. Licensing decreases with industry concentration. Within firms, higher industry concentration is associated with less licensing.

Fact 1 is borne out in Table 1: the extensive margin for licensing is low relative to R&D, yet mean licensing intensity is of the same order as mean R&D intensity (full-sample means: $\Pr(\text{Licensing} > 0) = 0.130$ vs. $\Pr(\text{R\&D} > 0) = 0.778$; intensities 0.214% vs. 0.305% of revenue). Age and size gradients align with a scaling interpretation: the licensing extensive margin and unconditional intensity rise with firm employment (from 0.108 and 0.175% for ≤ 5 employees to 0.458 and 2.480% for > 100), while R&D is widespread throughout but also increases with size (extensive margin $0.773 \rightarrow 0.882$; intensity $0.203\% \rightarrow 1.145\%$). Because larger firms are disproportionately represented in R&D-intensive industries, the size gradients in Table 1 combine within-industry scaling with cross-industry

Table 1: Extensive margins and mean intensities relative to revenue

	Share with > 0		Mean intensity	
	Licensing	R&D	Licensing	R&D
Full sample	0.130	0.778	0.214	0.305
By age				
0–2	0.102	0.776	0.178	0.411
3–5	0.120	0.783	0.221	0.313
6–10	0.126	0.789	0.214	0.351
11–15	0.137	0.783	0.319	0.286
> 15	0.149	0.770	0.174	0.224
By employment				
≤ 5	0.108	0.773	0.175	0.203
6–10	0.154	0.768	0.225	0.439
11–50	0.191	0.801	0.229	0.460
51–100	0.270	0.837	2.272	1.890
> 100	0.458	0.882	2.480	1.145

Notes: Intensities are sample means of licensing income or R&D expenditure divided by gross output (revenue), expressed in percent.

composition. We therefore treat Table 1 as descriptive. In the panel regressions below, firm fixed effects absorb time-invariant differences in firms' industry affiliation and baseline innovativeness, so the coefficients are identified from within-firm time variation in R&D intensity and in the industry-level competition measure.

Facts 2–3 are assessed in within-firm regressions of licensing intensity on R&D intensity and industry normalized concentration index HHI, which we present in form $1 - \text{HHI}$ so that it can be read as indication of competitive pressure:

$$\text{Licensing}_{it} = \beta_1\, \text{R\&D}_{it} + \beta_2\, (1 - \text{HHI}_{j(i)t}) + \beta' X_{it} + \gamma_t + \alpha_i + \varepsilon_{it}, \tag{1}$$

where i indexes firms and j 3-digit industries, X_{it} includes log total assets, log equity, and log employment, γ_t are year fixed effects, and α_i are firm fixed effects. Variables are shares of revenue; standard errors are clustered at the firm level. To accommodate zeros and approximate log responses, we also estimate specifications using the inverse hyperbolic sine (IHS) transformation on licensing, R&D intensity, and competitiveness.

The central observation is twofold: licensing scales with R&D and increases under greater competitive pressure. In levels, a one–percentage-point increase in the R&D share is associated with a 0.405–0.424 percentage-point increase in the licensing share (cols. 1–2), and the competition coefficient turns statistically significant once firm fixed effects absorb time-invariant heterogeneity (from 0.010 to 0.035; cols. 1–2). In IHS form, coefficients allow an elasticity interpretation for non-small values: the elasticity of licensing with respect to R&D is about 0.166, and with respect to competition about 0.026 (cols. 3–4). These patterns are robust in: the subsample of licensing firms, the subsample of R&D performers, and R&D-intensive sectors; Manufacturing (C), Information and Communication (J), and Professional, Scientific & Technical Activities (M).

Table 2: Baseline vs IHS

Specification:	Baseline		IHS	
Model:	(1)	(2)	(3)	(4)
Coefficients				
R&D expenditure	0.405	0.424	0.226	0.166
	(0.145)	(0.154)	(0.066)	(0.092)
(1-HHI)	0.010	0.035	0.000	0.026
	(0.008)	(0.011)	(0.004)	(0.009)
Details				
Additional controls	Yes	Yes	Yes	Yes
Year Fixed-effects	Yes	Yes	Yes	Yes
Firm Fixed-effects		Yes		Yes
Fit statistics				
Observations	297,153	297,153	297,153	297,153
R^2	0.088	0.622	0.034	0.490
Within R^2	0.088	0.098	0.033	0.012

Notes: Clustered standard errors at firm level in parentheses. Additional controls include total assets, equity and employment in logs.

These facts are descriptive and provide quantitative targets and discipline for the modeling exercise: licensing is concentrated yet material, scales with innovative effort, and is stronger under greater competitive pressure. Especially the observation about the industry's competition situation and licensing is important as it suggests that licensing is not neutral to changes in competition. In the firm level fixed effects the change in competitive pressure appears to correlate with higher tendency of licensing.

3 Endogenous Growth Model with Licensing Option

The economy is composed of a continuum of intermediate goods that are inputs in the production of a final good consumed by the representative household. In each intermediate good market two incumbent firms engage in Bertrand competition to gain the monopoly of production. Intermediate firms use labor as their only input and they are heterogeneous in productivity. The productivity develops in a step-by-step fashion (the quality ladder) as in Grossman and Helpman (1991). Firms invest in R&D activity which allows them to catch up to the technology frontier or increase the existing gap to laggard firms. To study how government subsidy on licensing affects growth and welfare by increasing knowledge-diffusion we craft Arrowian dynamics in our model (Arrow, 1963). In a given intermediate good market the technology leader may not be able to extract monopoly profit because of the Bertrand competition. The extraction of monopoly profit is possible only if the price that monopoly would set is smaller than the marginal cost of the laggard firm which means that the technology cap must be large enough. If the technology cap is relatively small, so that the leader is not able to charge the monopoly price, then it can be possible to increase the profit by licensing the more advanced technology to the laggard firm and sharing profits appropriately. However, while this increases the flow profit of the technology leader, it also increases the chance of being outrun by the laggard firm. Thus, a balancing of risk and payoff must be considered.

3.1 Preferences of the Representative Household

The time in our model is discrete. There is a representative consumer, or household, with utility function

$$U_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{s-t} \ln(C_s). \tag{2}$$

Here C_t is consumption at time t and $\rho > 0$ is the discount rate. The periodic budget constraint of the household is

$$C_t + A_t = w_t L_t + (1 + r_{t-1}) A_{t-1} + G_t, \tag{3}$$

where L_t is the labor supply, w_t the wage, A_t the amount of assets, r_t the interest rate, and G_t the amount of government transfers. Labor is supplied inelastically ($L_t = 1$), but divided between the R&D and production sectors. Households own all the firms in the economy. Thus, asset market clearing condition implies that total assets must equal the value of all firms. Let $j \in [0, 1]$ index markets, or equivalently leader–follower pairs. In pair j, leader and follower have values $V_t^L(j)$ and $V_t^F(j)$. Aggregate assets are thus

$$A_t = \int_0^1 \left(V_t^L(j) + V_t^F(j) \right) \mathrm{d}j. \tag{4}$$

3.2 Market for the Final Good

The final good that is used solely for consumption is produced in a perfectly competitive market. The amount of final good in the economy is obtained from the amount of intermediate goods used in production by Dixit-Stiglitz-aggregation (Dixit & Stiglitz, 1977);

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\sigma-1}{\sigma}} \mathrm{d}j \right)^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where y_{jt} is the amount of intermediate good $j \in [0, 1]$ produced at time t, and σ is the time invariant elasticity of substitution of intermediate products. This is where our model differs from Akcigit and Ates (2019, 2023)[13]; they use Cobb-Douglas aggregation for the final good.

Aggregate profit from supplying the final good at time t, when the price of the final good is P_t and the price of the intermediate good j is p_{jt}, can be written as;

$$P_t Y_t - \int_0^1 p_{jt} y_{jt} \mathrm{d}j = P_t \left(\int_0^1 y_{jt}^{\frac{\sigma-1}{\sigma}} \mathrm{d}j \right)^{\frac{\sigma}{\sigma-1}} - \int_0^1 p_{jt} y_{jt} \mathrm{d}j. \tag{6}$$

[13]Akcigit and Ates (2019, 2023) use a different model of competition for intermediate good market in the working paper version Akcigit and Ates (2019) and the published version Akcigit and Ates (2023). We use the model of the working paper version.

Thus, the final good profit maximizing expenditure on each intermediate good j is;

$$p_{jt}y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{1-\sigma} Z_t, \tag{7}$$

where $Z_t = P_t Y_t$ is the spending on final good at time t and from now on we normalize the consistent price index $P_t = \left[\int_0^1 p_{jt}^{1-\sigma}\mathrm{d}j\right]^{\frac{1}{1-\sigma}}$ as $P_t \equiv 1$.

3.3 Markets for Intermediate Goods

There is a unit mass of intermediate-good lines indexed by $j \in [0,1]$. Each line hosts two incumbent firms indexed by $i \in \{1,2\}$ (with the other firm denoted by $-i \equiv 3 - i$). Firm i in line j produces a perfectly substitutable variety with linear technology

$$y_{ijt} = q_{ijt} l_{ijt},$$

where q_{ijt} is productivity and l_{ijt} is labor.

For each line j and time t, define the productivity-ordered pair

$$q_{jt} \equiv \max\{q_{1jt}, q_{2jt}\}, \qquad z_{jt} \equiv \min\{q_{1jt}, q_{2jt}\},$$

and the (ratio) technology gap $\Delta_{jt} \equiv q_{jt}/z_{jt} \geq 1$. We refer to the firm with productivity q_{jt} as the leader and the other as the follower. In neck-and-neck states, $\Delta_{jt} = 1$ and the two firms are symmetric.

Variables without a firm index refer to line-level outcomes. In particular,

$$y_{jt} = y_{1jt} + y_{2jt}, \qquad l_{jt} = l_{1jt} + l_{2jt}.$$

Away from neck-and-neck states ($\Delta_{jt} > 1$), only the leader produces; in neck-and-neck states ($\Delta_{jt} = 1$), the two firms split production symmetrically.

Although only one of the firms is often active at a time, it may not be able to extract the monopoly rent from the market. With linear technology, the marginal cost of a firm with productivity q is;

$$mc_t(q) = \frac{w_t}{q}.$$

The unconstrained monopoly price of a producer with productivity q is

$$p_{jt}^M = \frac{\sigma}{\sigma - 1}\, mc_t(q) = \frac{\sigma}{\sigma - 1}\frac{w_t}{q}.$$

Under Bertrand competition with perfect substitutes, the follower's marginal cost caps the price at

$$p_{jt}^B = mc_t(z) = \frac{w_t}{z_{jt}}.$$

Hence, absent licensing, the equilibrium price is

$$p_{jt} = \min\left\{p^M_{jt},\, p^B_{jt}\right\} = \min\left\{\frac{\sigma}{\sigma-1}\frac{w_t}{q_{jt}},\, \frac{w_t}{z_{jt}}\right\} = \min\left\{\frac{\sigma}{\sigma-1},\, \Delta_{jt}\right\}\frac{w_t}{q_{jt}}. \tag{8}$$

Since the marginal cost of the laggard inactive firm is $mc(z_{jt}) > mc(q_{jt})$, the monopoly price of the leader can be lower than the marginal cost of the follower. If this is the case, then despite the Bertrand competition, the leader can extract monopoly rents. For this to happen q_{jt} must be sufficiently higher than z_{jt}. On the other hand, if the monopoly price is higher than the marginal cost of the laggard inactive firm, then Bertrand competition brings the price down to marginal cost of the laggard firm and monopoly rent cannot be extracted.

Licensing incentives. Under the observation that technology gap dictates the possibility of monopoly price under Bertrand competition, we now describe the central incentives for using licensing. The technology leader can have a *static incentive* to use licensing; allow the laggard firm to produce alone (to set the monopoly price) and extract part of this revenue through a licensing fee. There is, of course, a *dynamic cost* for doing this; the laggard firm can more easily take the position of a technology leader. Moreover, there is also more subtle *dynamic incentive* to use licensing, the leader can extract also rents from the potentially increased option value of the follower. This is a formal representation of what we mean by Arrowian dynamics and becomes more explicit below when we define the firms' values.

Output and labor demand in production. The output of a product line and its labor demand can be solved by maximizing the profit $\pi_{jt} = p_{jt}y_{jt} - w_t l_{jt}$ under (7). The product line output is;

$$y_{jt} = \begin{cases} \left(\Delta_{jt}\frac{w_t}{q_{jt}}\right)^{-\sigma} Y_t, & \text{if } \Delta_{jt} < \frac{\sigma}{\sigma-1}, \\ \left(\frac{\sigma}{\sigma-1}\frac{w_t}{q_{jt}}\right)^{-\sigma} Y_t, & \text{if } \Delta_{jt} \geq \frac{\sigma}{\sigma-1}. \end{cases}$$

and the labor demand directly follows from $l_{jt} = y_{jt}/q_{jt}$. Notice that when $\Delta_{jt} = 1$ the output and labor are split equally.

Flow profits. The leader's flow profit when it produces is either follower capped profit π^B or "monopoly price"-profit π^M:

$$\pi_{jt} = \begin{cases} \pi^B_{jt} = \frac{\Delta_{jt}-1}{\Delta_{jt}^{\sigma}}\left(\frac{w_t}{q_{jt}}\right)^{1-\sigma} Y_t, & \text{if } \Delta_{jt} < \frac{\sigma}{\sigma-1}, \\ \pi^M_{jt} = \frac{\frac{\sigma}{\sigma-1}-1}{\left(\frac{\sigma}{\sigma-1}\right)^{\sigma}}\left(\frac{w_t}{q_{jt}}\right)^{1-\sigma} Y_t, & \text{if } \Delta_{jt} \geq \frac{\sigma}{\sigma-1}. \end{cases}$$

When the leader licenses, the fee is a within-period transfer and the contract entails a deductible fixed cost κ for each party. We therefore distinguish firm-level operating profits in the licensing regime:

$$\pi^{CL}_{jt} \equiv F_t(q_{jt}, z_{jt}) - \kappa, \qquad \pi^{CF}_{jt} \equiv \pi^M(q_{jt}, z_{jt}) - F_t(q_{jt}, z_{jt}) - \kappa,$$

where C indicates a contracted state and superscripts L and F refer to the leader and follower. For aggregation and government revenues, define licensing-set operating profits (net of deductible contracting costs) as total profits in the line:

$$\pi_{jt}^{C} \equiv \pi_{jt}^{CL} + \pi_{jt}^{CF} = \pi^{M}(q_{jt}, z_{jt}) - 2\kappa.$$

Henceforth, π_{jt} denotes the leader's flow profit when it produces (i.e. either π_{jt}^{B} or π_{jt}^{M}), and for the licensing state the profits are written with $F_t(q, z)$ for clarity, and we define it below.

3.4 R&D Behavior and Innovation Dynamics

The R&D behavior and innovation dynamics follow Akcigit and Ates (2023) with minor modifications that are necessary to accommodate the licensing behavior. Firms invest in R&D to increase their technological level. Let us denote the R&D expenditure of firm i in market j at time t by R_{ijt}. This cost comes from hiring labor h_{ijt}. The amount needed to generate the arrival rate x_{ijt} for a new innovation is given by the equation

$$R_{ijt} = \alpha \frac{x_{ijt}^{\gamma}}{\gamma} q_{ijt} w_t. \tag{9}$$

Here γ is the inverse elasticity of R&D with respect to workforce and w_t is the wage rate of labor in the economy at time t. We use the productivity scaled R&D cost function as in Akcigit and Kerr (2018). This captures the observation that ideas are getting harder to find (see e.g. Bloom, Jones, van Reenen, and Webb (2020)). From this we can derive the amount of labor h_{ijt} needed to generate the arrival rate x_{ijt} as

$$x_{ijt} = \left(\frac{\gamma}{\alpha} \frac{R_{ijt}}{q_{ijt} w_t} \right)^{\frac{1}{\gamma}} = \left(\frac{\gamma}{\alpha} \frac{h_{ijt}}{q_{ijt}} \right)^{\frac{1}{\gamma}}. \tag{10}$$

Firms' productivity evolves successively step-by-step as innovations accumulate (aka the quality ladder). Initially the productivity of both firms is 1 implying $\Delta_{jt} = 1$. Thus, because of the Bertrand competition, the profit of both firms is 0. This is called the *neck-and-neck* situation. Both firms in the market invest in R&D to generate the arrival rates x_{ijt} and x_{-ijt}. If a firm innovates successfully at time t, then its productivity increases by a factor λ at time $t+1$, i.e.

$$q_{ij(t+1)} = \lambda q_{ijt}, \tag{11}$$

where λ is a random variable with a support $(1, \infty)$. When the follower innovates it may become the leader if the innovation step is large enough.[14] We denote the density function of productivity step λ by $f(\cdot)$, and the cumulative distribution function by $F(\cdot)$. In addition, it is possible that a new product will replace the market of an existing product altogether. To model this we assume that in any market j there is a potential entrant that innovates to replace the market with a new product. This entrant generates the arrival

[14]Notice that in Akcigit and Ates (2023) the follower can never take the position of a leader in one innovation step; at best the market can transition to a neck-and-neck situation.

rate of a new product using equation (9) with $\alpha = \alpha_e$. Let us denote the amount of labor hired by the potential entrant in market j by h_{ej} and the arrival rate generated by x_{ej}. Once born, this new market starts from a neck-and-neck situation with productivity of both firms equal to the average effective productivity of the economy i.e.

$$q_{ijt} = q_{-ijt} = \int_0^1 q_{jt}\,\mathrm{d}j \equiv Q(t).$$

Acemoglu and Cao (2015) use the term *imitative entry* to describe this.

We use the following timing for these events. Between times t and $t+1$ we first look whether the market is replaced with a new product or not. For market j this happens with probability x_{ej}. Notice that this arrival rate is not really market dependent; it only depends on the expectation of the average productivity at the next period. Thus we can write $x_{ej} = x_{et}$. If the innovation effort of the entrant fails, the innovator among the leader and follower is selected by an exogenous fair coin toss. Thus, at most one innovation can materialize per period at each market.

4 Towards Equilibrium Dynamics: Behavior of the Firms

The Euler equation of our model implies that

$$\frac{1+r_t}{1+\rho} = 1 + g_t$$

must hold at all times, where r_t is the interest rate, g_t is the growth rate of the output, and ρ is the time discount rate of the representative household.

A firm's continuation value depends on: whether the leader produces or licenses, whether the firm is the leader or the follower, and the productivity levels on its product line. We therefore introduce state notation to keep track of these dimensions. Let $(q, z) \in \mathbb{R}^2_+$ with $q \geq z$ denote leader and follower productivities. A firm's role is $d \in \{F, L\}$, and neck-and-neck corresponds to $q = z$. Let $k \in \{P, C\}$ denote production versus licensing (C for contract, not to mix with L). The state is (q, z, d, k), with (d, k) written as superscripts.

At time t, the leaders value when producing with our notation is $V_t^{LP}(q, z)$ and the value of a leader that is licensing is $V_t^{LC}(q, z)$. Thus, the beginning of period value function of the leader can be written as;

$$V_t^L(q, z) = \max\{V_t^{LP}(q, z),\, V_t^{LC}(q, z)\}.$$

The follower's continuation value depends on whether the leader licenses. Let $V_t^{FC}(q, z)$ denote the follower's value under licensing (when it produces), and let $V_t^{FP}(q, z)$ denote the follower's value when the leader produces (so the follower is inactive). Then

$$V_t^F(q, z) = \begin{cases} V_t^{FC}(q, z), & \text{if licensing is used at } (q, z), \\ V_t^{FP}(q, z), & \text{otherwise.} \end{cases}$$

In neck-and-neck, both firms have value $V_t(q)$.

4.1 Fee Determination and Value of the Licensing Contract

A licensing contract is negotiated each period and specifies a one-period fee $F_t(q,z)$ paid by the follower to the leader. We model fee determination as generalized Nash bargaining over the surplus from licensing, taking continuation values and the innovation environment as given. The fee is a within-period transfer and therefore does not directly affect marginal R&D incentives. Signing the contract entails a fixed contracting cost 2κ per period (one cost κ borne by each party). We treat this cost as a deductible business expense, so it reduces after-tax payoffs by $(1-\tau)\kappa$ for each party.

If no agreement is reached at (q,z), the leader produces and the follower is inactive. We denote the resulting continuation values by

$$D_t^L(q,z) \equiv V_t^{LP}(q,z), \qquad D_t^F(q,z) \equiv V_t^{FP}(q,z). \tag{12}$$

If a license is signed, the follower produces as a monopolist using the licensed technology and pays the fee $F_t(q,z)$ to the leader. Let π_{jt}^M denote the follower's within-period monopoly operating profit under using leaders technology alone (before paying the fee). We impose the cash-flow feasibility restriction $0 \le F_t(q,z) \le \pi_{jt}^M - \kappa$. We assume a proportional profit tax at rate τ_t and that licensing fees are taxable income for the leader and deductible expenses for the follower; hence the fee shifts *after-tax* payoffs by $(1-\tau_t)F_t$.

Let $\bar{V}_t^{LC}(q,z)$ and $\bar{V}_t^{FC}(q,z)$ denote, respectively, the leader's and follower's continuation values under licensing when the fee at period t is set to zero (holding fixed the licensing transition structure). Then the fee enters continuation values linearly, and the joint licensing value is independent of the fee:

$$V_t^{LC}(q,z;F) = \bar{V}_t^{LC}(q,z) + (1-\tau_t)F, \qquad V_t^{FC}(q,z;F) = \bar{V}_t^{FC}(q,z) - (1-\tau_t)F, \tag{13}$$

$$J_t^C(q,z) \equiv V_t^{LC}(q,z;F) + V_t^{FC}(q,z;F) = \bar{V}_t^{LC}(q,z) + \bar{V}_t^{FC}(q,z). \tag{14}$$

The fee $F_t(q,z)$ solves the generalized Nash bargaining problem

$$F_t(q,z) \in \arg\max_F \left(V_t^{LC}(q,z;F) - D_t^L(q,z)\right)^{\delta} \left(V_t^{FC}(q,z;F) - D_t^F(q,z)\right)^{1-\delta}, \tag{15}$$

subject to participation ($V_t^{LC} \ge D_t^L$ and $V_t^{FC} \ge D_t^F$) and feasibility ($0 \le F \le \pi_{jt}^M - \kappa$) constraints. Using (13), it is convenient to define the gains from licensing at zero fee,

$$\Delta_t^L(q,z) \equiv \bar{V}_t^{LC}(q,z) - D_t^L(q,z), \qquad \Delta_t^F(q,z) \equiv \bar{V}_t^{FC}(q,z) - D_t^F(q,z).$$

The unconstrained Nash fee is then

$$F_t^N(q,z) = \frac{\delta\, \Delta_t^F(q,z) - (1-\delta)\, \Delta_t^L(q,z)}{1-\tau_t}. \tag{16}$$

Finally, we impose the participation and feasibility constraints by clipping $F_t^N(q,z)$ to the admissible interval. In particular, the leader's participation requires $F \geq -\Delta_t^L(q,z)/(1-\tau_t)$, the follower's participation requires $F \leq \Delta_t^F(q,z)/(1-\tau_t)$, and feasibility requires $F \leq \pi_{jt}^M - \kappa$, together with $F \geq 0$. The implemented fee $F_t(q,z)$, therefore, differs from $F_t^N(q,z)$ only by satisfying the imposed constraints.

4.2 Value of Firms

Market leader. The value function of a leader that does not use licensing is defined by the following program:

$$
\begin{aligned}
V_t^{LP}(q,z) = \max_{x_{qzt}} \Bigg\{ & (1-\tau_t)\pi_{jt} - (1-s_t)\alpha \frac{x_{qzt}^{\gamma}}{\gamma} q w_t \\
& + (1-x_{et}) \frac{1}{1+r_t} \Bigg\{ \tfrac{1}{2} \left[x_{qzt} \int_1^{\infty} V_{t+1}^L(\lambda q, z) f(\lambda)\, d\lambda + (1-x_{qzt}) V_{t+1}^L(q,z) \right] \\
& + \tfrac{1}{2} \Bigg[x_{-qzt} \left(\int_1^{q/z} V_{t+1}^L(q, \lambda z) f(\lambda)\, d\lambda + \int_{q/z}^{\infty} V_{t+1}^F(\lambda z, q) f(\lambda)\, d\lambda \right) \\
& + (1-x_{-qzt}) V_{t+1}^L(q,z) \Bigg] \Bigg\} \Bigg\}. \qquad (17)
\end{aligned}
$$

Here x_{qzt} (x_{-qzt}) is the arrival rate generated by the optimal R&D investment decision of the leader (follower). The first term under the maximization is the operating profit (with follower capped or monopoly price) after the deduction of the corporate tax (rate τ_t). The second term is the amount of spending on R&D that is needed to generate the arrival rate x_{qzt} (subsidized at rate s_t; this can be interpreted as the R&D investment tax credit that is used in almost every country). The third large term represents the increase in the leader's market value in the next period. This depends on three possible mutually exclusive events; the market is replaced by a new product, the follower innovates instead of the leader, or the leader itself innovates.

To formulate the value function of a leader under licensing we specify the key mechanism that can generate positive social surplus from the licensing: the diffusion of knowledge. We do this by assuming an exogenous probability that determines whether the follower innovates on its own productivity or on the leader's productivity. With probability η the follower innovates on the leader's productivity, and thus immediately takes the leading position, and with probability $(1-\eta)$ the follower innovates on its own productivity. The follower can leapfrog the leader even if it innovates on its own productivity. The only difference is that the probability is lower in this case. The value function of a leader that is licensing is then defined by the following program:

$$
\begin{aligned}
V_t^{LC}(q,z) = \max_{x_{qzt}} \Bigg\{ & (1-\tau_t)(F_t(q,z)-\kappa) - (1-s_t)\alpha \frac{x_{qzt}^{\gamma}}{\gamma} q w_t \\
& + (1-x_{et}) \frac{1}{1+r_t} \Bigg\{ \tfrac{1}{2} \left[x_{qzt} \int_1^{\infty} V_{t+1}^{L}(\lambda q, z) f(\lambda) d\lambda + (1-x_{qzt}) V_{t+1}^{L}(q,z) \right] \\
& + \tfrac{1}{2} \Bigg[x_{-qzt} \Bigg(\eta \int_1^{\infty} V_{t+1}^{F}(\lambda q, q) f(\lambda) d\lambda + (1-\eta) \int_1^{q/z} V_{t+1}^{L}(q, \lambda z) f(\lambda) d\lambda \\
& + (1-\eta) \int_{q/z}^{\infty} V_{t+1}^{F}(\lambda z, q) f(\lambda) d\lambda \Bigg) + (1-x_{-qzt}) V_{t+1}^{L}(q,z) \Bigg] \Bigg\} \Bigg\}
\end{aligned} \tag{18}
$$

Now the leader faces a larger probability of losing the leading market position if the follower, that is now active after licensing, innovates. The first integral in the third row represents the case where the licensing firm becomes the follower firm because the rival incumbent innovates on its productivity. The higher risk of becoming the follower is compensated by higher operating profits, through the fee $F_t(q,z)$, that are not possible without taking the risk. Note also that the leader fully controls whether the licensing state is chosen.

The follower. The value of a laggard firm that is not producing under a license is defined by the following program:

$$
\begin{aligned}
V_t^{FP}(q,z) = \max_{x_{-qzt}} \Bigg\{ & -(1-s_t)\alpha \frac{x_{-qzt}^{\gamma}}{\gamma} z w_t \\
& + (1-x_{et}) \frac{1}{1+r_t} \Bigg\{ \tfrac{1}{2} \left[x_{qzt} \int_1^{\infty} V_{t+1}^{F}(\lambda q, z) f(\lambda)\, d\lambda + (1-x_{qzt}) V_{t+1}^{F}(q,z) \right] \\
& + \tfrac{1}{2} \Bigg[x_{-qzt} \left(\int_1^{q/z} V_{t+1}^{F}(q, \lambda z) f(\lambda)\, d\lambda + \int_{q/z}^{\infty} V_{t+1}^{L}(\lambda z, q) f(\lambda)\, d\lambda \right) \\
& + (1-x_{-qzt}) V_{t+1}^{F}(q,z) \Bigg] \Bigg\} \Bigg\}.
\end{aligned} \tag{19}
$$

The operating profit is missing from this equation because the laggard firm is inactive when it is not producing under a license. Nevertheless, the continuation value of the firm is positive if it continues to invest in R&D; it might eventually be able to capture the position of a market leader. The term $(1-x_{et})$ at the beginning of the second row comes again from the fact that a new product can replace the market entirely.

On the other hand, if the laggard firm is producing under a license, then its value is defined by the program:

$$
\begin{aligned}
V_t^{FC}(q,z) = \max_{x_{-qzt}} \Bigg\{ & (1-\tau_t)[\pi^M(q,z) - F_t(q,z) - \kappa] - (1-s_t)\alpha \frac{x_{-qzt}^{\gamma}}{\gamma} z w_t \\
& + (1-x_{et})\frac{1}{1+r_t}\Bigg\{ \tfrac{1}{2}\left[x_{qzt}\int_1^{\infty} V_{t+1}^F(\lambda q, z) f(\lambda) d\lambda + (1-x_{qzt}) V_{t+1}^F(q,z) \right] \\
& + \tfrac{1}{2}\Bigg[x_{-qzt}\Bigg(\eta \int_1^{\infty} V_{t+1}^L(\lambda q, q) f(\lambda) d\lambda + (1-\eta)\int_1^{q/z} V_{t+1}^F(q, \lambda z) f(\lambda) d\lambda \\
& + (1-\eta)\int_{q/z}^{\infty} V_{t+1}^L(\lambda z, q) f(\lambda) d\lambda \Bigg) + (1-x_{-qzt}) V_{t+1}^F(q,z) \Bigg] \Bigg\} \Bigg\}
\end{aligned} \tag{20}
$$

Now the follower is responsible for producing and can collect the monopoly profit since the leader abstains from producing. Thus, operating profit is equal to monopoly profit minus the licensing fee and contracting cost. This can be positive or negative as the follower is paying also for the increased possibility of leapfrogging, but in practice we focus to cash-flow-feasible cases in the baseline. However, this increased possibility of leapfrogging still affects the outcome of bargaining over the fee.

Neck-and-neck position. In our model the probability that an existing market with a positive technology cap will transition into a neck-and-neck position is zero (a zero measure event). However, each new market starts from a neck-and-neck position with two firms that have average productivity. The value of firms in a neck-and-neck position is defined by the program:

$$
\begin{aligned}
V_t(q) = \max_{x_{qt}} \Bigg\{ & -(1-s_t)\alpha \frac{x_{qt}^{\gamma}}{\gamma} q w_t \\
& + (1-x_{et})\frac{1}{1+r_t}\Bigg[\tfrac{1}{2}\left(x_{qt}\int_1^{\infty} V_{t+1}^L(\lambda q, q) f(\lambda)\, d\lambda + (1-x_{qt}) V_{t+1}(q) \right) \\
& + \tfrac{1}{2}\left(x_{-qt}\int_1^{\infty} V_{t+1}^F(\lambda q, q) f(\lambda)\, d\lambda + (1-x_{-qt}) V_{t+1}(q) \right) \Bigg] \Bigg\}.
\end{aligned} \tag{21}
$$

Here x_{-qt} is the optimal arrival rate of the competitor. Since firms are symmetric in a neck-and-neck position, the equality $x_{qt} = x_{-qt}$ must hold at equilibrium. Now the operating profit is missing because of the Bertrand competition. Notice that, in contrast to markets with a positive productivity cap, both firms get an equal probability to innovate.

Entrant. In every market the potential entrant selects the arrival rate x_{et} that solves the following program:

$$
\max_{x_{et}} \left\{ -(1-s_t)\alpha_e \frac{x_{et}^{\gamma}}{\gamma} Q(t) w_t + \frac{1}{1+r_t} x_{et} E\Big[V_{t+1}(Q(t+1), Q(t+1)) \Big] \right\}
$$

The entrant selects the arrival rate based on the expected value of a neck-and-neck position with average productivity $Q(t+1)$ at the next period $t+1$.

From this point onward we work with normalized firm values. For any continuation-value object $V_t^X(q,z)$, with $X \in \{L, LP, LC, F, FP, FC\}$, define

$$\tilde{q} \equiv \frac{q}{Q(t)}, \qquad \tilde{z} \equiv \frac{z}{Q(t)}, \qquad V_t^X(q,z) = Q(t)\, v_t^X(\tilde{q},\tilde{z}). \tag{22}$$

In the neck-and-neck case we write $V_t(q) = Q(t)\, v_t(\tilde{q})$ with $\tilde{q} \equiv q/Q(t)$. Let g denote the balanced-growth-path (BGP) growth rate of average productivity $Q(t)$, so that $Q(t+1) = (1+g)Q(t)$. Since the final-good technology is homothetic, output Y_t grows at rate g along the BGP as well.

Lemma. The normalized value functions $v_t^X(\tilde{q},\tilde{z})$ satisfy the same Bellman programs as (17)–(21) after dividing through by $Q(t)$ and expressing all objects in normalized units. In particular, replace

$$w_t \mapsto \tilde{w}_t \equiv \frac{w_t}{Q(t)}, \qquad Y_t \mapsto \tilde{Y}_t \equiv \frac{Y_t}{Q(t)}, \qquad F_t(q,z) \mapsto \tilde{F}_t(\tilde{q},\tilde{z}) \equiv \frac{F_t(q,z)}{Q(t)},$$

and write flow profits in normalized form as $\pi(q,z,w_t,Y_t) = Q(t)\,\pi(\tilde{q},\tilde{z},\tilde{w}_t,\tilde{Y}_t) = \tilde{\pi}_{jt}$. Moreover, along the BGP the Euler equation implies $(1+r) = (1+\rho)(1+g)$, so the discount factor becomes $\frac{1}{1+r} = \frac{1}{(1+\rho)(1+g)}$. Consequently, any next-period continuation value term transforms as

$$\frac{1}{1+r_t} V_{t+1}^X(q',z') = \frac{Q(t)}{1+\rho}\, v_{t+1}^X\left(\frac{q'}{Q(t+1)}, \frac{z'}{Q(t+1)}\right) = \frac{Q(t)}{1+\rho}\, v_{t+1}^X\left(\frac{\tilde{q}'}{1+g}, \frac{\tilde{z}'}{1+g}\right),$$

where $\tilde{q}' \equiv q'/Q(t)$ and $\tilde{z}' \equiv z'/Q(t)$ denote next-period productivities expressed in units of $Q(t)$.

At the BGP, Equation (17), for example, becomes

$$\begin{aligned} v_t^{LP}(\tilde{q},\tilde{z}) = \max_{x_{\tilde{q}\tilde{z}t}} \Bigg\{ & (1-\tau)\tilde{\pi} - (1-s_t)\alpha \frac{x_{\tilde{q}\tilde{z}t}^{\gamma}}{\gamma} \tilde{q}\tilde{w}_t \\ & + (1-x_{et}) \frac{1}{1+\rho} \Bigg[\tfrac{1}{2} \Bigg(x_{\tilde{q}\tilde{z}t} \int_1^{\infty} v_t^L\Big(\frac{\lambda \tilde{q}}{1+g}, \frac{\tilde{z}}{1+g}\Big) f(\lambda)\, d\lambda + (1-x_{\tilde{q}\tilde{z}t})\, v_t^L\Big(\frac{\tilde{q}}{1+g}, \frac{\tilde{z}}{1+g}\Big)\Bigg) \\ & + \tfrac{1}{2} \Bigg(x_{-\tilde{q}\tilde{z}t} \Bigg[\int_1^{\tilde{q}/\tilde{z}} v_t^L\Big(\frac{\tilde{q}}{1+g}, \frac{\lambda\tilde{z}}{1+g}\Big) f(\lambda)\, d\lambda + \int_{\tilde{q}/\tilde{z}}^{\infty} v_t^F\Big(\frac{\lambda\tilde{z}}{1+g}, \frac{\tilde{q}}{1+g}\Big) f(\lambda)\, d\lambda \Bigg] \\ & + (1-x_{-\tilde{q}\tilde{z}t})\, v_t^L\Big(\frac{\tilde{q}}{1+g}, \frac{\tilde{z}}{1+g}\Big)\Bigg)\Bigg]\Bigg\}. \end{aligned} \tag{23}$$

Proof. Using $V_t^X(q,z) = Q(t) v_t^X(\tilde{q},\tilde{z})$ and $Q(t+1) = (1+g)Q(t)$, we have

$$\begin{aligned} \frac{1}{1+r_t} \frac{V_{t+1}^X(q',z')}{Q(t)} &= \frac{1}{(1+\rho)(1+g)} \frac{V_{t+1}^X(q',z')}{Q(t)} = \frac{1}{1+\rho} \frac{V_{t+1}^X(q',z')}{Q(t+1)} \\ &= \frac{1}{1+\rho}\, v_{t+1}^X\left(\frac{q'}{Q(t+1)}, \frac{z'}{Q(t+1)}\right), \end{aligned}$$

which yields the stated normalization of continuation terms. Dividing the original programs (17)–(21) by $Q(t)$ and substituting $\tilde{w}_t = w_t/Q(t)$, $\tilde{Y}_t = Y_t/Q(t)$, and $\tilde{F}_t = F_t/Q(t)$

delivers the normalized programs. □

4.3 Optimal R&D Investment Decisions

Programs (17)–(21) imply closed-form policy rules for optimal R&D effort (innovation arrival probabilities). It is convenient to state these rules first in levels, as functions of unnormalized continuation values $V_t(\cdot)$ and the frontier $Q(t)$. The normalized rules used in the computation follow immediately by the same scaling argument as in previous section.

Incumbent innovation. Fix a line with ordered productivities (q, z), $q \geq z$, and let x_{qzt} and x_{-qzt} denote the leader's and follower's innovation probabilities. Under our power cost function, the optimal interior choice satisfies

$$x_{qzt} = \left[\frac{(1 - x_{et})}{2(1 - s_t)\alpha\, q\, w_t\, (1 + \rho)} \left(\int_1^\infty V_{t+1}^L(\lambda q, z)\, f(\lambda)\, d\lambda - V_{t+1}^L(q, z) \right) \right]^{\frac{1}{\gamma - 1}}, \tag{24}$$

irrespective of whether licensing is used.

If the follower is inactive (i.e. the leader produces), the follower's optimal innovation probability is

$$x_{-qzt} = \left[\frac{(1 - x_{et})}{2(1 - s_t)\alpha\, z\, w_t\, (1 + \rho)} \left(\int_1^{q/z} V_{t+1}^F(q, \lambda z)\, f(\lambda)\, d\lambda \right.\right.$$
$$\left.\left. + \int_{q/z}^\infty V_{t+1}^L(\lambda z, q)\, f(\lambda)\, d\lambda - V_{t+1}^F(q, z) \right) \right]^{\frac{1}{\gamma - 1}}. \tag{25}$$

If the follower produces under a licensing contract, then conditional on a successful follower innovation, the follower innovates on the leader's technology with probability η. The follower's optimal innovation probability becomes

$$x_{-qzt} = \left[\frac{(1 - x_{et})}{2(1 - s_t)\alpha\, z\, w_t\, (1 + \rho)} \left(\eta \int_1^\infty V_{t+1}^L(\lambda q, q)\, f(\lambda)\, d\lambda \right.\right.$$
$$+ (1 - \eta) \int_1^{q/z} V_{t+1}^F(q, \lambda z)\, f(\lambda)\, d\lambda$$
$$\left.\left. + (1 - \eta) \int_{q/z}^\infty V_{t+1}^L(\lambda z, q)\, f(\lambda)\, d\lambda - V_{t+1}^F(q, z) \right) \right]^{\frac{1}{\gamma - 1}}. \tag{26}$$

Since licensing is determined by the state (here summarized by the productivity ratio $\Delta = q/z$), the equilibrium policy is single-valued: for any given (q, z), either (25) or (26) applies, depending on whether licensing is used at that state.

Neck-and-neck. There is no licensing in neck-and-neck states. If $q = z$, the common innovation probability is

$$x_{qt} = \left[\frac{(1-x_{et})}{2(1-s_t)\alpha\, w_t\,(1+\rho)} \left(\int_1^{\infty} V^L_{t+1}(\lambda q, q)\, f(\lambda)\, d\lambda - V_{t+1}(q) \right) \right]^{\frac{1}{\gamma-1}}. \tag{27}$$

Entry. The entrant's optimal innovation probability is

$$x_{et} = \left[\frac{V_{t+1}(Q(t+1))}{(1-s_t)\alpha\, w_t\,(1+\rho)} \right]^{\frac{1}{\gamma-1}}, \tag{28}$$

where $V_{t+1}(Q(t+1))$ is the value of a newly created line (which starts at the frontier).

Normalized policies can then be recovered from these in the following way. Along the BGP, write $V_t^X(q,z) = Q(t)\, v_t^X(\tilde{q},\tilde{z})$ with $\tilde{q} = q/Q(t)$, $\tilde{z} = z/Q(t)$, and similarly $w_t = Q(t)\tilde{w}_t$. Substituting these relations into (24)–(28) yields the normalized policy rules, with the additional mapping $q' = q/(1+g)$ and $z' = z/(1+g)$ for next-period normalized states.

4.4 Evolution of the Productivity Distribution

At time t, each line $j \in [0,1]$ is characterized by the productivity-ordered pair (q_{jt}, z_{jt}), with $q_{jt} \geq z_{jt}$. The collection $\{(q_{jt}, z_{jt})\}_{j\in[0,1]}$ is equivalently summarized by the cross-sectional distribution $\mathcal{M}_t$ over states $s_t \equiv (q_t, z_t) \in \mathbb{R}^2_+$.

Line-level dynamics are discrete-time Markov. Conditional on the current state $s = (q,z)$, next period's state is drawn from a transition kernel $P_t(s,\cdot)$. We define the kernel through its action on test functions: for any bounded measurable φ,

$$(P_t\varphi)(q,z) \equiv \mathbb{E}[\varphi(q_{t+1}, z_{t+1}) \mid (q_t, z_t) = (q,z)]\,.$$

Conditional on no entry, an exogenous fair coin toss selects which incumbent gets the opportunity to attempt innovation. Let x_e denote the entry probability, let $x_{qz,t}$ and $x_{-qz,t}$ denote the leader's and follower's innovation probabilities in state (q,z) at time t, and let $\lambda > 1$ be the innovation step with density $f(\lambda)$. Let $I_t(q,z) \in \{0,1\}$ indicate whether licensing is used at (q,z).

Write P_t^P for the no-licensing (leader-produces) operator and P_t^C for the licensing operator. Then

$$P_t\varphi(q,z) = \big(1 - I_t(q,z)\big)\, P_t^P\varphi(q,z) + I_t(q,z)\, P_t^C\varphi(q,z). \tag{29}$$

The operators are:

$$
\begin{aligned}
(P_t^P\varphi)(q,z) =& x_{e,t}\,\varphi(Q(t+1),Q(t+1)) \\
&+(1-x_{e,t})\Bigg\{\frac{1}{2}\Big[(1-x_{qz,t})\,\varphi(q,z)+x_{qz,t}\int_1^\infty \varphi(\lambda q,z)f(\lambda)\,d\lambda\Big] \\
&\qquad+\frac{1}{2}\Big[(1-x_{-qz,t})\,\varphi(q,z) \\
&\qquad+x_{-qz,t}\int_1^\infty \varphi\big(\operatorname{ord}(q,\lambda z)\big)f(\lambda)\,d\lambda\Big]\Bigg\}.
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
(P_t^C\varphi)(q,z) =& x_{e,t}\,\varphi(Q(t+1),Q(t+1)) \\
&+(1-x_{e,t})\Bigg\{\frac{1}{2}\Big[(1-x_{qz,t})\,\varphi(q,z)+x_{qz,t}\int_1^\infty \varphi(\lambda q,z)f(\lambda)\,d\lambda\Big] \\
&\qquad+\frac{1}{2}\Big[(1-x_{-qz,t})\,\varphi(q,z)+x_{-qz,t}\int_1^\infty \Big(\eta\,\varphi(\lambda q,q) \\
&\qquad+(1-\eta)\,\varphi\big(\operatorname{ord}(q,\lambda z)\big)\Big)f(\lambda)\,d\lambda\Big]\Bigg\}.
\end{aligned}
\tag{31}
$$

where $\operatorname{ord}(a,b)\equiv(\max\{a,b\},\min\{a,b\})$ enforces productivity ordering, and η is the probability that (under licensing) a successful follower innovation builds on the leader's technology.

Given P_t, the distribution evolves as

$$
\mathcal{M}_{t+1}(A)=\int_{\mathbb{R}_+^2} P_t(s,A)\,\mathcal{M}_t(ds) \quad \text{for all Borel } A\subset\mathbb{R}_+^2.
$$

On the BGP, $Q(t+1)=(1+g)Q(t)$ and equilibrium policies are time-invariant functions of the normalized state $\tilde{s}=(\tilde{q},\tilde{z})\equiv(q/Q(t),z/Q(t))$. Define the scaling map $S_t(q,z)=(q/Q(t),z/Q(t))$ and let μ_t be the distribution of $\tilde{s}$. The induced normalized kernel $\tilde{P}$ is time-homogeneous and satisfies, for any φ,

$$
(\tilde{P}\varphi)(\tilde{s})\equiv\mathbb{E}[\varphi(\tilde{s}_{t+1})\mid\tilde{s}_t=\tilde{s}] \quad \text{with} \quad \tilde{s}_{t+1}=S_{t+1}\big(s_{t+1}\big),\; s_{t+1}\sim P_t\big(S_t^{-1}(\tilde{s}),\cdot\big).
$$

Equivalently, $\mu_{t+1}=\mu_t\tilde{P}$. The stationary BGP distribution μ solves $\mu=\mu\tilde{P}$. For numerical work we implement $\tilde{P}$ directly.

4.5 Competitive equilibrium and balanced growth path

Fix policy sequences $\{\tau_t,s_t,s_{e,t}\}_{t\geq0}$ and an initial cross-sectional distribution $\mathcal{M}_0$ over line-level states $s\equiv(q,z)\in\mathbb{R}_+^2$ with $q\geq z$. A competitive equilibrium is a collection of sequences

$$
\Big\{C_t,A_t,w_t,r_t,G_t\Big\}_{t\geq0},\quad \Big\{I_t(\cdot),F_t(\cdot),x_{e,t},x_{qz,t}(\cdot),x_{-qz,t}(\cdot)\Big\}_{t\geq0},\quad \{\mathcal{M}_t\}_{t\geq0},
$$

together with line-level allocations (production labor and R&D labor) such that: Given $\{w_t, r_t, G_t\}$, the representative household chooses $\{C_t, A_t\}$ to maximize (2) subject to (3). For each state (q, z) and date t, incumbent firms' value functions and policy rules solve the value functions in Section 4.2. Intermediate-good prices are set according to the pricing rule implied by the chosen regime. Net transfers satisfy the government budget constraint for all t, and G_t is rebated to households as a lump-sum transfer. Wage w_t is such that labor market clearing holds, the goods market clears with $C_t = Y_t$, and the asset market clears with household assets equal to the aggregate value of firms. The cross-sectional distribution evolves under the equilibrium-induced transition kernel P_t, where P_t is constructed from the equilibrium policies $\{I_t, F_t, x_{e,t}, x_{qz,t}, x_{-qz,t}\}$.

A BGP is then defined as a competitive equilibrium in which there exists a constant g and an aggregate quality index $Q(t)$ such that $Q(t+1) = (1+g)Q(t)$, and, under the normalizations defined above, the equilibrium is stationary: all normalized value functions, policy rules, prices, and the cross-sectional distribution of normalized states are time-invariant.

5 Aggregation and Market Clearing

We solve the model by tracking the evolution of the cross-sectional distribution of line-level states relative to the frontier index $Q(t)$. Two distinct objects matter. First, the *state distribution* summarizes the technological configuration in each line: at time t line j is characterized by the productivity-ordered pair (q_{jt}, z_{jt}) (leader and follower), and the cross section is summarized by the distribution of these pairs. Second, *only the producing technology* contributes directly to goods-market aggregates at a point in time, but the non-producing technology affects aggregates indirectly through: the Bertrand price gap, and future innovation and licensing incentives. Our normalization index $Q(t)$ is an average over *leader technologies*:

$$Q(t) \equiv \int_0^1 q_{jt} \, \mathrm{d}j. \tag{32}$$

Let M_t denote the set of lines in which the leader produces and charges the monopoly price, L_t the set of lines in which a licensing contract is used, and B_t the remaining lines in which Bertrand competition pins the price at the follower's marginal cost.[15] Define the *effective productivity* in line j by

$$\hat{q}_{jt} \equiv \left(\frac{\sigma - 1}{\sigma}\right) q_{jt} \, \mathbf{1}\{j \in M_t \cup L_t\} + z_{jt} \, \mathbf{1}\{j \in B_t\}.$$

This object captures the fact that in $M_t \cup L_t$ prices are set at a constant markup over the leader's marginal cost, while in B_t they are capped by the follower's marginal cost. Using demand curve and firms' optimal pricing, the equilibrium wage can be written as a

[15]The set B_t also includes neck-and-neck lines, since the pricing outcome is identical. By construction $M_t \cup L_t \cup B_t = [0, 1]$.

function of the cross section:

$$w_t = \left[\int_0^1 \hat{q}_{jt}^{\sigma-1}\,\mathrm{d}j\right]^{\frac{1}{\sigma-1}} = \left[\int_{M_t \cup L_t} \left(\frac{\sigma-1}{\sigma} q_{jt}\right)^{\sigma-1} \mathrm{d}j + \int_{B_t} (z_{jt})^{\sigma-1}\,\mathrm{d}j\right]^{\frac{1}{\sigma-1}}. \tag{33}$$

Output is determined by (5) and labor market clearing holds at all times:

$$1 = \int_0^1 \Big(l_{jt} + h_{jt} + h_{-jt} + h_{ejt}\Big)\,\mathrm{d}j, \tag{34}$$

where l_{jt} is production labor in line j, h_{jt} and h_{-jt} are R&D labor used by the leader and follower, and h_{ejt} is entrant R&D labor.

Product-market clearing is $C_t = Y_t$. Using the household budget constraint and firms' accounting, it is convenient to express this in terms of production labor and operating profits. Let $H_t^{RD} \equiv \int_0^1 (h_{jt} + h_{-jt} + h_{ejt})\,\mathrm{d}j$ denote total R&D labor and let Π_t denote aggregate operating profits (net of production wages, before paying R&D wages). Then goods-market clearing can be written as

$$Y_t = w_t\big(1 - H_t^{RD}\big) + \Pi_t + G_t, \tag{35}$$

where G_t is the net transfer from the government.

The government taxes corporate operating profits at rate τ_t and subsidizes direct R&D at rate s_t. Let π_{jt}^M and π_{jt}^B denote operating profits in the monopoly-pricing and Bertrand-pricing sets. In licensing lines, the fee F_t is an intra-line transfer, so total operating profits equal

$$\pi_{jt}^C \equiv \pi_{jt}^{LC} + \pi_{jt}^{FC} = \pi^M(q, z) - 2\kappa,$$

where κ is a deductible contracting cost incurred by each party. Net government revenues are

$$\begin{aligned} G_t = &\int_{M_t} \Big[\tau_t \pi_{jt}^M - s_t w_t \frac{\alpha}{\gamma}(x_{jt}^\gamma q_{jt} + x_{-jt}^\gamma z_{jt})\Big] dj \\ &+ \int_{B_t} \Big[\tau_t \pi_{jt}^B - s_t w_t \frac{\alpha}{\gamma}(x_{jt}^\gamma q_{jt} + x_{-jt}^\gamma z_{jt})\Big] dj \\ &+ \int_{L_t} \Big[\tau_t \pi_{jt}^C - s_t w_t \frac{\alpha}{\gamma}(x_{jt}^\gamma q_{jt} + x_{-jt}^\gamma z_{jt})\Big] dj \\ &- s_{e,t} w_t \frac{\alpha_e}{\gamma} x_{e,t}^\gamma. \end{aligned} \tag{36}$$

A positive G_t is rebated to households as a lump-sum transfer, and a negative G_t is financed by a lump-sum tax.

Along the BGP, $Q(t+1) = (1+g)Q(t)$ and equilibrium objects scale with $Q(t)$. Let $\tilde{q}_{jt} \equiv q_{jt}/Q(t)$ and $\tilde{z}_{jt} \equiv z_{jt}/Q(t)$. Then denote

$$w_t = Q(t)\tilde{w}_t, \qquad Y_t = Q(t)\tilde{Y}_t, \qquad G_t = Q(t)\tilde{G}_t, \tag{37}$$

where the normalized aggregates depend only on the stationary distribution of normalized states and the associated regime partition (M_t, L_t, B_t) induced by the licensing rule. In particular, (33) implies

$$\tilde{w}_t = \left[\int_0^1 \tilde{\tilde{q}}_{jt}^{\sigma-1}\, \mathrm{d}j\right]^{\frac{1}{\sigma-1}}, \qquad \tilde{\tilde{q}}_{jt} \equiv \left(\frac{\sigma-1}{\sigma}\right) \tilde{q}_{jt}\, \mathbf{1}\{j \in M_t \cup L_t\} + \tilde{z}_{jt}\, \mathbf{1}\{j \in B_t\}.$$

Since $\tilde{w}_t$ and $\tilde{Y}_t$ are constant on the BGP, both w_t and Y_t grow at rate g.

Let μ denote the stationary BGP distribution of normalized states $(\tilde{q}, \tilde{z})$, with $\tilde{q} \geq \tilde{z}$, and define $\Delta \equiv \tilde{q}/\tilde{z}$. Observe that the expected innovation step size is

$$E[\lambda] \equiv \int_1^\infty \lambda f(\lambda)\, d\lambda.$$

Write $I(\tilde{q}, \tilde{z}) \in \{0, 1\}$ for the licensing indicator and note that the innovation opportunity is assigned by a fair coin toss, so each incumbent is selected with probability $1/2$.

Proposition. On the BGP, the gross growth rate admits the decomposition

$$g = g^L + g^{F,\text{diff}} + g^{F,\text{leap}},$$

where

$$g^L = \frac{1}{2} \int \tilde{x}_{\tilde{q}\tilde{z}}\, \tilde{q}\, \big(E[\lambda] - 1\big)\, \mu(d\tilde{q}, d\tilde{z}), \tag{38}$$

$$g^{F,\text{diff}} = \frac{1}{2} \int \tilde{x}_{-\tilde{q}\tilde{z}}\, I(\tilde{q}, \tilde{z}) \eta\, \tilde{q}\, \big(E[\lambda] - 1\big)\, \mu(d\tilde{q}, d\tilde{z}), \tag{39}$$

$$g^{F,\text{leap}} = \frac{1}{2} \int \tilde{x}_{-\tilde{q}\tilde{z}}\, \big(1 - I(\tilde{q}, \tilde{z})\eta\big) \left[\int_\Delta^\infty (\lambda \tilde{z} - \tilde{q})\, f(\lambda)\, d\lambda\right] \mu(d\tilde{q}, d\tilde{z}), \tag{40}$$

with $\Delta \equiv \tilde{q}/\tilde{z}$. The term g^L is growth from leader innovations. The term $g^{F,\text{diff}}$ is growth from follower innovations that, under licensing, build on the leader's technology with probability η (the "shoulders"/diffusion mechanism), so that the innovation step λ is applied to $\tilde{q}$ rather than to $\tilde{z}$. The term $g^{F,\text{leap}}$ is growth from follower innovations on the follower's own technology (which occurs with probability $1 - I(\tilde{q}, \tilde{z})\eta$); these contribute to growth only when the follower leapfrogs the leader, i.e. when $\lambda > \Delta$.

Proof. By definition of $Q(t)$ in (32), the stationary normalization satisfies

$$\int \tilde{q}\, \mu(d\tilde{q}, d\tilde{z}) = 1.$$

Entry resets a destroyed line to $(Q(t+1), Q(t+1))$ and hence to $(1, 1)$ in normalized units. Because $Q(t)$ is defined as the cross-sectional average of leader productivities, this reset does not directly affect the mean condition; growth is driven by the expected change in the leader productivity of surviving lines. Therefore,

$$1 + g = \int \mathbb{E}\big[\tilde{q}' \mid (\tilde{q}, \tilde{z})\big]\, \mu(d\tilde{q}, d\tilde{z}),$$

where $\tilde{q}'$ denotes next period's leader productivity measured in units of $Q(t)$ (i.e. before dividing by $1 + g$).

Conditional on the state $(\tilde{q}, \tilde{z})$, a fair coin toss selects the leader or follower to attempt innovation. If the leader is selected, a successful innovation multiplies q by λ, so the expected increment in $\tilde{q}$ is $\tilde{x}_{\tilde{q}\tilde{z}}\, \tilde{q}\, (E[\lambda] - 1)$. If the follower is selected, a successful innovation either builds on the leader's technology with probability $I(\tilde{q}, \tilde{z})\eta$, in which case the post-innovation pair is $(\lambda q, q)$ and the leader increment is $\tilde{q}(E[\lambda] - 1)$, or else builds on the follower's technology, in which case the leader increases only when the follower leapfrogs. The expected leapfrogging increment is

$$\tilde{x}_{-\tilde{q}\tilde{z}}\, \mathbb{E}[\max\{\tilde{q}, \lambda\tilde{z}\} - \tilde{q}] = \tilde{x}_{-\tilde{q}\tilde{z}} \int_{\Delta}^{\infty} (\lambda\tilde{z} - \tilde{q})\, f(\lambda)\, d\lambda,$$

where we used that $\max\{\tilde{q}, \lambda\tilde{z}\} = \tilde{q}$ for $\lambda \leq \Delta$ and $\max\{\tilde{q}, \lambda\tilde{z}\} = \lambda\tilde{z}$ for $\lambda > \Delta$. Combining this with the licensing branch (probability $I(\tilde{q}, \tilde{z})\eta$) and then weighting by the coin-toss probabilities $1/2$ and integrating over μ delivers the decomposition above. □

It is useful to relate our normalization index $Q(t)$ to the standard CES "quality index" implicit in the final-good aggregator, and to separate frontier technology from the quality actually embodied in production. Define the *produced CES quality index*

$$Q_t^Y \;\equiv\; \left(\int_0^1 \hat{q}_{jt}^{\sigma-1}\, \mathrm{d}j\right)^{\frac{1}{\sigma-1}}, \tag{41}$$

where $\hat{q}_{jt}$ is the effective productivity that determines the intermediate-good price in line j. The final-good firm's FOCs imply $w_t = Q_t^Y$, so (41) is the relevant production-side quality index.

Next, define the *frontier CES index* (the CES index that would prevail if every line could price at the monopoly markup based on the leader technology) as

$$Q_t^F \;\equiv\; \left(\int_0^1 \left(\frac{\sigma-1}{\sigma} q_{jt}\right)^{\sigma-1} \mathrm{d}j\right)^{\frac{1}{\sigma-1}}. \tag{42}$$

The ratio

$$\Omega_t \;\equiv\; \frac{Q_t^Y}{Q_t^F} \;\in [1, \infty) \tag{43}$$

summarizes the wedge between the frontier embodied in production and the frontier technology available in the economy. Intuitively, $\Omega_t > 1$ whenever a positive measure of lines is in B_t, because Bertrand competition pins prices at the follower's marginal cost, which implies $z_{jt} \geq (\frac{\sigma-1}{\sigma})q_{jt}$ on those lines; thus the term $(\frac{\sigma-1}{\sigma} q_{jt})^{\sigma-1}$ is replaced by the weakly larger $z_{jt}^{\sigma-1}$. Finally, our average-quality normalization $Q(t)$ in (32) is a linear (technology) moment of the full state distribution, while Q_t^Y and Q_t^F are CES moments that emphasize the upper tail; these indices coincide up to a constant only in knife-edge cases (e.g. degenerate distributions), but they move together on the BGP since all scale with $Q(t)$.

Table 3: List of Parameter Values

A. Externally Calibrated		B. Internally Calibrated	
Parameter	Value	Parameter	Value
ρ	0.05	σ	6.78
τ	0.2	α	0.401
l_s	0	α_e	19.7
s	0.12	η	0.222
γ	2.86	ϕ	28.9
ψ	0.5	κ	0.021

6 Quantifying the Role of Licensing for Growth and Welfare

6.1 Calibration

The balanced growth path (BGP) of the model economy is characterized by the parameter vector $\Theta \equiv \{\rho, \sigma, \tau, s, \gamma, \alpha, \alpha_e, \eta, \phi, \delta, \kappa\}$. ϕ governs the distribution of innovation step sizes. We set the household discount rate at $\rho = 0.05$. Following Akcigit and Ates (2023), the curvature of the R&D production function is $\gamma = 1/0.35$.[16] We set the corporate income tax rate at 20% and the baseline R&D subsidy to 12%, consistent with the values of the OECD for profitable firms in 2024 (Source: OECD Data Explorer). The licensing subsidy is set to zero, reflecting the absence of such a program in Finland during the sample period. Finally, we set $\delta = 0.5$, implying equal bargaining power for firms.

After these external choices, the remaining parameters are $\Gamma \equiv \{\alpha, \alpha_e, \eta, \phi, \sigma, \kappa\}$, which we calibrate jointly to match a set of empirical moments. While point identification of each parameter in isolation is not available, the selected moments[17] load most strongly on particular parameters through first-order effects: we choose α_e to match an annual firm entry rate of 7%, discipline ϕ using a long-run average labor productivity growth rate of 0.94% per year, target α to attain an R&D-to-GDP ratio of 3.7%, use η to match a licensing-fees-to–value-added ratio of 0.7%, use κ to match extensive margin of licensing to 13%, and use σ to match the average markup rate of 9% in Finland (Bighelli, di Mauro, Melitz, and Mertens (2023)). Despite joint determination in equilibrium, this strategy provides useful empirical discipline because each target primarily moves the corresponding parameter along the dimension of its first-order effect, yielding reasonably sharp identification in practice.

6.2 Quantitative Impact of the Licensing Option

This section quantifies the role of licensing by comparing the baseline equilibrium (licensing allowed and endogenously chosen in a subset of states) to a counterfactual in which licensing is shut down.[18]

[16] An alternative, $\gamma = 1/0.5$, could be used based on Acemoglu, Akcigit, Bloom, and Kerr (2018).

[17] The moments originate from Statistics Finland when available directly. For licensing we calculate the numbers from the microdata directly as official record does not exist.

[18] Throughout, policy parameters (s, s_e, l_s) and all primitives other than the licensing option are held fixed. In the no-licensing counterfactual we set $I(\tilde{q}, \tilde{z}) \equiv 0$ and solve for the new stationary equilibrium.

Table 4: Model Fit

Moment	Model (%)	Data (%)	Source
M1 TFP growth	0.863	0.94	Statistics Finland
M2 R&D to GDP	3.53	3.7	Statistics Finland
M3 Firm entry	7.43	7	Statistics Finland
M4 Markup	9	9	Bighelli et. al (2021)
M5 Licensing fee share (intensive)	0.711	0.711	Own calculation from microdata
M6 Licensing share (extensive)	10	13	Own calculation from microdata

Table 5 Panel A summarizes the main aggregate outcomes. Allowing licensing raises the balanced-growth rate from 0.833% to 0.863%, but lowers the level of output and wages (from $Y = 0.988$ to $Y = 0.978$, and from $w = 0.952$ to $w = 0.939$). The welfare implication is therefore not pinned down by growth alone. In the baseline calibration, shutting down licensing increases consumption-equivalent welfare by 0.362%.

The growth decomposition in Table 5 Panel B shows that in the baseline the follower innovation contributes to growth significantly, but the diffusion through licensing is quantitatively small in the baseline calibration ($g^{F,\text{diff}}/g = 0.018$). In contrast, the level loss is consistent with reduced surplus from product-market distortions. In particular, the profit share Π/Y is higher under licensing (7.41% vs. 6.83%), while the CES wedge measure $\Omega \equiv Q/Q_L$ indicates that the produced-quality index Q sits further above the monopoly-frontier index Q_L when licensing is shut down ($\Omega = 1.08$ vs. 1.07). Together these patterns point to a first-order tradeoff: licensing modestly increases long-run growth via diffusion and follower incentives, but reallocates surplus through pricing regimes in a way that reduces the level of consumption in the baseline calibration.

Table 5 Panel C connects these aggregate effects to line-level equilibrium structure. Licensing is used in 10.1% of lines, and the average technology gap in licensing states is small ($q/z \simeq 1.02$), consistent with licensing being most attractive when firms are close (and the static motive to relax Bertrand pressure is strongest). Shutting down licensing shifts mass into Bertrand lines (from 56% to 66%) and reduces incumbent and entrant R&D (Table 5 Panel A), with the largest decline in incumbent effort. This is visible both in total R&D labor (from 3.53% to 3.29%) and in the entry rate (from 7.43% to 7.19%).

Licensing also changes the distribution of markups. Average markups fall slightly when licensing is shut down (from 1.09 to 1.08), and markup dispersion falls (sd from 0.0656 to 0.0625). This combination is consistent with licensing increasing the prevalence of states in which prices are effectively pinned to the follower cap and/or with a reallocation across pricing regimes. Finally, the leapfrogging rate declines when licensing is shut down (3.06% to 2.61%), and the licensing-specific diffusion channel disappears by construction.

Licensing affects the economy primarily through innovation incentives. Allowing licensing raises total R&D labor from 3.29% to 3.53%, with most of the increase coming from incumbents (incumbent R&D rises from 2.91% to 3.12%). Consistent with the model mechanism, the extra effort is concentrated in licensing states, which feature small technol-

Table 5: Licensing shut down.

Panel A: Main aggregates

	Baseline	No licensing
Growth g (%)	0.863	0.833
CE welfare gain vs. baseline (%)	0.000	0.362
Output Y (level)	0.978	0.988
Wage w (level)	0.939	0.952
Profit share Π/Y (%)	7.41	6.83
Markup (revenue-weighted)	1.09	1.08
Entry rate x_E (%)	7.43	7.19
R&D labor share (%)	3.53	3.29
Incumbent R&D (%)	3.12	2.91
Entrant R&D (%)	0.411	0.374
Growth per R&D (g/H^{RD})	0.245	0.253

Panel B: Growth decomposition and diffusion

	Baseline	No licensing
g^L/g	0.792	0.805
$g^{F,\text{diff}}/g$	0.018	0.000
$g^{F,\text{leap}}/g$	0.191	0.195
Leapfrog rate (%)	3.06	2.61
Follower success prob. (%)	11.1	10.4

Panel C: Market structure, technology gaps, and dispersion

	Baseline	No licensing
Status share: Licensing (%)	10.1	0.0
Status share: Monopoly (%)	19.4	19.7
Status share: Bertrand (%)	56.0	66.0
Status share: Neck-and-neck (%)	14.5	14.3
Avg gap q/z in Licensing	1.02	–
Avg gap q/z in Monopoly	1.34	1.34
Avg gap q/z in Bertrand	1.07	1.07
Markup dispersion (sd)	0.0656	0.0625
Frontier CES index Q_L	0.880	0.882
Quality CES index Q	0.939	0.952
Wedge $\Omega \equiv Q/Q_L$	1.07	1.08

Notes: H^{RD} is total R&D labor as a share of total labor. "Growth per R&D" reports g/H^{RD} using g and H^{RD} in percentage points (a unit-free ratio).

ogy gaps ($q/z \simeq 1.02$) and therefore high marginal returns to diffusion and rent-sharing. Despite the higher R&D intensity, the growth gain in the baseline calibration is modest (from 0.833% to 0.863%), implying a slightly lower growth-per-R&D ratio under licensing. This pattern helps rationalize why welfare can increase when licensing is shut down: the dynamic benefit from higher innovation is partially offset by the level cost induced by the licensing-related change in pricing regimes.

The baseline calibration implies a welfare loss from allowing licensing, but the sign can reverse when diffusion is strong. Table 6 reports an extreme variant with $\eta = 1$, where licensing raises growth substantially (from 0.833% to 0.931%) and now *increases* welfare (shutting down licensing yields a CE loss of 0.455%). In this case diffusion through li-

Table 6: Licensing shut down (high diffusion, $\eta = 1$).

Panel A: Main aggregates

	Baseline (max η)	No licensing
Growth g (%)	0.931	0.833
CE welfare gain vs. baseline (%)	0.000	-0.455
Output Y (level)	0.973	0.988
Wage w (level)	0.933	0.952
Profit share Π/Y (%)	7.70	6.83
Markup (revenue-weighted)	1.10	1.08
Entry rate x_E (%)	7.43	7.19
R&D labor share (%)	3.76	3.29
Incumbent R&D (%)	3.35	2.91
Entrant R&D (%)	0.410	0.374
Growth per R&D (g/H^{RD})	0.248	0.253

Panel B: Growth decomposition and diffusion

	Baseline (max η)	No licensing
g^L/g	0.730	0.805
$g^{F,\text{diff}}/g$	0.135	0.000
$g^{F,\text{leap}}/g$	0.135	0.195
Leapfrog rate (%)	5.39	2.61
Follower success prob. (%)	12.2	10.4

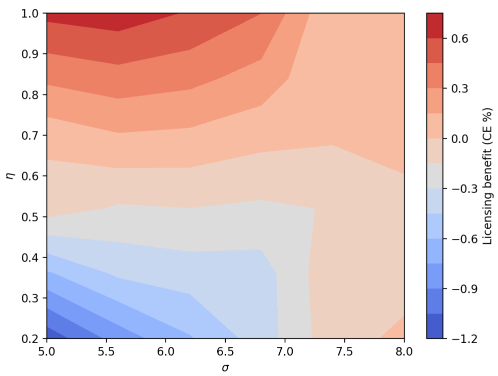
Panel C: Market structure, technology gaps, and dispersion

	Baseline (max η)	No licensing
Status share: Licensing (%)	15.1	0.0
Status share: Monopoly (%)	20.0	19.7
Status share: Bertrand (%)	50.5	66.0
Status share: Neck-and-neck (%)	14.5	14.3
Avg gap q/z in Licensing	1.02	–
Avg gap q/z in Monopoly	1.33	1.34
Avg gap q/z in Bertrand	1.08	1.07
Markup dispersion (sd)	0.0657	0.0625
Frontier CES index Q_L	0.879	0.882
Quality CES index Q	0.933	0.952
Wedge $\Omega \equiv Q/Q_L$	1.06	1.08

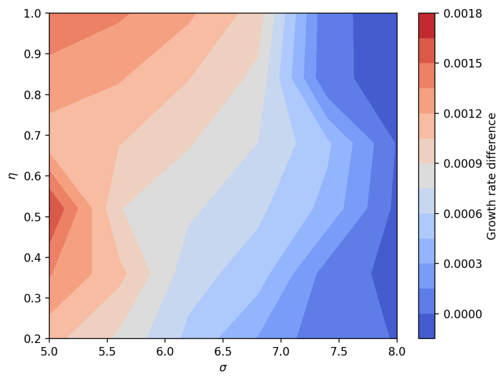
Notes: H^{RD} is total R&D labor as a share of total labor. "Growth per R&D" reports g/H^{RD} using g and H^{RD} in percentage points (a unit-free ratio).

censing becomes quantitatively meaningful ($g^{F,\text{diff}}/g = 0.135$) and leapfrogging is much higher, consistent with licensing raising the follower's option value and thereby stimulating innovation.

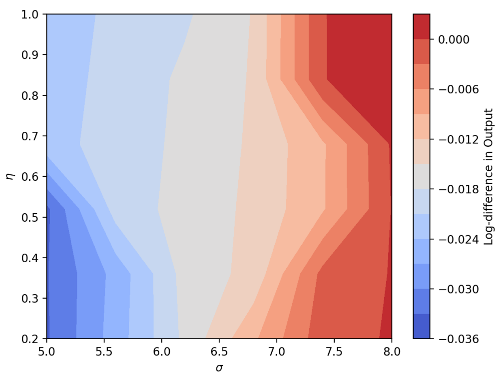
To map the central tradeoff, Figure 1 varies product substitutability σ and diffusion intensity η while holding other parameters fixed. The welfare effect of licensing is increasing in η and decreasing in σ. Intuitively, higher η amplifies the dynamic benefit of licensing through diffusion and follower incentives, whereas higher σ compresses monopoly rents and weakens the static motive to license. The same forces appear in the corresponding changes in growth (Figure 1b) and output levels (Figure 1c): diffusion-rich environments generate growth gains large enough to offset the level costs, while highly substitutable



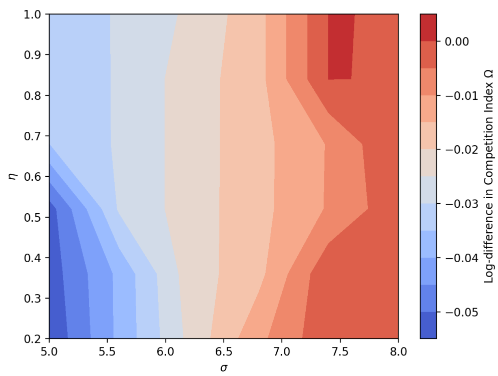
(a) Welfare.



(b) Growth rate.



(c) Output level.



(d) Competition Index Ω.

Figure 1: Effect of allowing licensing, relative to shutting it down, over: welfare, growth, and output across (σ, η).

environments deliver small growth gains and a predominantly level-driven welfare effect.

A final implication of the bargaining structure is that higher leapfrogging risk is not necessarily detrimental for the leader: by raising the follower's continuation value, it can increase the joint surplus from contracting and (through Nash bargaining) increase the leader's ability to extract rents via the licensing fee. This observation can be seen by comparing the share of products in licensing state between Table 5 and 6.

6.3 Innovation Policies for Licensing

We close the quantitative analysis by asking how innovation policy should respond when licensing is an available (and endogenous) margin. The policy instruments are: an incumbent R&D subsidy s that applies symmetrically to both incumbents in every line, and an additional licensing-contingent R&D subsidy ℓ_s paid to incumbents *only* in states in which a licensing contract is signed.[19] The licensing-contingent subsidy is additive: in licensing states, an incumbent receives the effective rate $s + \ell_s$, whereas in non-licensing states it receives s.

We proceed in two steps. First, we choose s to maximize welfare subject to $\ell_s = 0$. Second, we allow the policymaker to jointly choose (s, ℓ_s). Table 7 reports the resulting equilibria for the baseline diffusion calibration (left block) and for the polar case $\eta = 1$ (right block).

Across both diffusion environments, the welfare-maximizing incumbent subsidy is substantially higher than the calibrated benchmark. In the baseline η economy, raising s from 12% to 47.9% increases the consumption-equivalent welfare measure by 0.939% (Panel A) and raises growth from 0.863% to 1.06%. The growth decomposition in Panel B indicates that the increase is broad-based: both leader and follower components rise as shares stay almost intact, while diffusion remains small in levels. The policy operates through the standard channel in innovation models: higher R&D subsidies raise incumbent effort (incumbent R&D rises from 3.12% to 5.81% of labor), thereby increasing innovation arrivals and, through equilibrium selection, raising the frequency of states in which incumbents have sufficient rents to invest (Panel C shows a shift toward monopoly lines).

A salient feature of the optimal-s counterfactual is that it raises growth while lowering the level of output and wages (Panel A). This is the familiar level–growth tension: subsidizing innovation reallocates labor from production into R&D and increases the long-run quality growth rate, but the contemporaneous resource cost depresses the level of goods production. In our environment, this reallocation is large, total R&D labor nearly doubles, so the output level falls even as the growth rate rises.

Allowing for an additional licensing-contingent subsidy ℓ_s produces only a modest incremental gain in the baseline η economy. The welfare improvement relative to the optimal-s policy is small (CE rises from 0.939% to 0.962%), and the growth rate is essentially unchanged (1.06% in both cases). Correspondingly, the optimal licensing-contingent

[19] We hold the entrant subsidy s_e fixed at its calibrated value. In an unreported one-dimensional search over s_e (holding the other instruments fixed), the welfare-maximizing value is close to zero. Given the negligible gains and to keep the policy experiment focused on the incumbent/licensing margin, we treat s_e as fixed.

Table 7: Welfare-maximizing subsidies with s_e fixed: baseline η vs. $\eta = 1$.

Panel A: Main aggregates

	Baseline η			Baseline $\eta = 1$		
	Baseline	Opt s ($\ell_s = 0$)	Opt s, ℓ_s	Baseline	Opt s ($\ell_s = 0$)	Opt s, ℓ_s
Growth g (%)	0.863	1.06	1.06	0.931	1.01	1.08
CE welfare gain vs. baseline (%)	0.000	0.939	0.962	0.000	0.570	0.808
Output Y (level)	0.978	0.950	0.951	0.973	0.964	0.952
Wage w (level)	0.939	0.935	0.935	0.933	0.938	0.931
Profit share Π/Y (%)	7.41	7.72	7.70	7.70	7.67	7.91
Markup (revenue-weighted)	1.09	1.09	1.09	1.10	1.09	1.10
Entry rate x_E (%)	7.43	7.70	7.68	7.43	7.51	7.59
R&D labor share (%)	3.53	6.26	6.19	3.76	5.12	5.81
Incumbent R&D (%)	3.12	5.81	5.74	3.35	4.69	5.38
Entrant R&D (%)	0.411	0.454	0.451	0.410	0.423	0.437
Growth per R&D $\left(g/H^{RD}\right)$	0.244	0.169	0.171	0.248	0.197	0.186

Panel B: Growth decomposition and diffusion

	Baseline η			Baseline $\eta = 1$		
	Baseline	Opt s ($\ell_s = 0$)	Opt s, ℓ_s	Baseline	Opt s ($\ell_s = 0$)	Opt s, ℓ_s
g^L/g	0.792	0.801	0.801	0.730	0.756	0.723
$g^{F,\text{diff}}/g$	0.018	0.016	0.017	0.135	0.096	0.148
$g^{F,\text{leap}}/g$	0.191	0.183	0.183	0.135	0.148	0.129
Leapfrog rate (%)	3.06	3.66	3.65	5.39	5.05	6.72
Follower success prob. (%)	11.1	13.5	13.5	12.2	13.2	14.8

Panel C: Market structure, technology gaps, and dispersion

	Baseline η			Baseline $\eta = 1$		
	Baseline	Opt s ($\ell_s = 0$)	Opt s, ℓ_s	Baseline	Opt s ($\ell_s = 0$)	Opt s, ℓ_s
Status share: Licensing (%)	10.1	9.13	9.14	15.1	9.84	14.4
Status share: Monopoly (%)	19.4	23.0	22.9	20.0	21.9	22.3
Status share: Bertrand (%)	56.0	54.5	54.5	50.5	54.4	49.4
Status share: Neck-and-neck (%)	14.5	13.4	13.4	14.5	13.9	13.9
Avg gap q/z in Licensing	1.02	1.02	1.02	1.02	1.02	1.02
Avg gap q/z in Monopoly	1.34	1.35	1.35	1.33	1.35	1.34
Avg gap q/z in Bertrand	1.07	1.08	1.08	1.08	1.08	1.08
Markup dispersion (sd)	0.0656	0.0658	0.0658	0.0657	0.0657	0.0655
Frontier CES index Q_L	0.880	0.879	0.879	0.879	0.881	0.879
Quality CES index Q	0.939	0.935	0.935	0.933	0.938	0.931
Wedge $\Omega \equiv Q/Q_L$	1.07	1.06	1.06	1.06	1.06	1.06

Panel D: Policy rates

	Baseline η			Baseline $\eta = 1$		
	Baseline	Opt s ($\ell_s = 0$)	Opt s, ℓ_s	Baseline	Opt s ($\ell_s = 0$)	Opt s, ℓ_s
Policy s (%)	12.0	47.9	47.5	12.0	35.0	38.0
Policy s_e (%)	12.0	12.0	12.0	12.0	12.0	12.0
Policy ℓ_s (%)	0.0	0.0	0.40	0.0	0.0	14.2

Notes: H^{RD} is total R&D labor as a share of total labor. "Growth per R&D" reports g/H^{RD} using g and H^{RD} in percentage points.

subsidy is close to zero (Panel B reports $\ell_s = 0.394\%$). Intuitively, when the diffusion channel is weak, licensing states are not the primary margin for aggregate innovation: the optimal policy primarily raises incentives economy-wide, and an additional state-contingent subsidy in the relatively small licensing region does not materially change equilibrium innovation behavior.

The role of the licensing-contingent subsidy changes sharply when $\eta = 1$, i.e. when follower innovation under licensing can fully build on the leader's technology. In this polar case, the joint policy (s, ℓ_s) generates a sizeable welfare gain (CE of 0.808%) and a larger growth response (growth rises from 0.931% to 1.08%). Unlike the baseline case, the optimal ℓ_s is economically meaningful (Panel D reports $\ell_s = 14.15\%$). Panel B shows that the additional growth is concentrated in the follower component as the leapfrogging rate rises from 5.39% to 6.72% and their success probability increases from 12.2% to 14.8%. This pattern is accompanied by larger share of growth coming from diffusion ($g^{F,\text{diff}}/g$ rises from 0.135 to 0.148 through 0.096), and is consistent with the central mechanism: when diffusion is technologically effective, subsidizing R&D specifically in licensing states amplifies the return to follower effort by increasing the probability that licensing-generated knowledge spillovers translate into frontier growth. In equilibrium, this targeted instrument also increases the prevalence of licensing (Panel B: licensing share rises from 15.1% to 14.4% after the joint optimization, compared to 9.84% under the policy that optimizes s alone), reinforcing the complementarity between diffusion and state-contingent innovation incentives.

In unreported comparative statics, we vary the elasticity of substitution σ and find that the marginal value of the licensing-contingent subsidy is larger when σ is high. When products are close substitutes, markups and licensing rents are compressed and licensing is used less frequently; in that region, a targeted subsidy that makes licensing states more attractive can meaningfully expand the measure of licensing and raise diffusion. Conversely, when σ is low, licensing is already used extensively through the static motive, and additional targeting yields limited gains. Across these exercises, the overall magnitudes are modest relative to the gains from raising the broad incumbent subsidy s, highlighting that the primary inefficiency is underinvestment in incumbent innovation rather than an underuse of licensing per se.

Two takeaways emerge. First, welfare-maximizing innovation policy calls for substantially higher incumbent R&D subsidies than the calibrated benchmark, reflecting strong dynamic spillovers from innovation. Second, the case for explicitly targeting the licensing margin depends on the strength of diffusion. When diffusion is weak, the optimal policy is essentially uniform across states; when diffusion is strong, a licensing-contingent subsidy becomes a useful complement that raises follower-driven growth.

7 Conclusion

Modern economic growth accelerated at the dawn of the Industrial Revolution, when practical and scientific knowledge began to accumulate and to be disseminated in ways not previously experienced (Mokyr, 2005). Two centuries later, growth is slowing in many advanced economies, and we still do not have a settled explanation for why. In the decades after World War II, some countries sustained growth rates that would today be considered exceptional, whereas many countries would now be satisfied with growth rates near 1%. One possibility is that ideas are becoming harder to find, so that maintaining a given growth rate requires ever-increasing research effort (Bloom et al., 2020). Another possibility is that the innovation system itself has evolved in ways that impede diffusion and entry, for example through strategic uses of intellectual property and enforcement that can stall competition (Cohen, Gurun, and Kominers (2019),Shapiro (2000)). Even if identifying the root cause is essential, a more modest and policy-relevant question is how to push the knowledge accumulation process back on track.

This paper studies one such margin: licensing of innovations from leaders to followers as a channel of knowledge diffusion, and the scope for policy to strengthen this channel. We develop an endogenous-growth general equilibrium model in which the technology leader chooses whether to license, trading off higher static rents against faster follower catch-up through diffusion. Quantitatively, the central message is that diffusion through licensing is not a free lunch. In the Finnish benchmark calibration, licensing improves diffusion and raises growth, yet it lowers consumption-equivalent welfare because the level effects of increased concentration dominate. At the same time, the same mechanism can raise welfare when diffusion through licensing is sufficiently strong, and the threshold depends sharply on product substitutability. This highlights that the welfare consequences of licensing depend on the interaction between the diffusion technology and the market structure consequences of licensing contracts.

Our policy analysis shows that optimal innovation policy changes once licensing is an endogenous margin. When the policymaker can use a uniform incumbent R&D subsidy together with an additional licensing-contingent subsidy, the optimal uniform subsidy is quantitatively important in all environments, reflecting the standard dynamic spillovers from innovation. Under baseline diffusion, however, the optimal licensing-contingent component is close to zero because the main welfare gains come from raising innovation incentives broadly, and stronger uniform R&D support tends to reduce the equilibrium prevalence of licensing by moving lines more quickly into monopoly-pricing states. When diffusion through licensing is strong, this interaction becomes first-order, and an additional licensing-contingent subsidy becomes a useful complement that sustains licensing and amplifies follower-driven growth through diffusion. Looking forward, three extensions appear particularly promising. First, more direct empirical discipline of diffusion through licensing is a priority, because it is central for the welfare ranking. Second, enriching the contracting environment (royalties, exclusivity, multiple licensees, and limits imposed by antitrust or standard-setting institutions) would sharpen the mapping from licensing to concentration and diffusion. Third, the framework can be used to evaluate targeted policy designs that are common in practice, including conditioning public R&D support on licensing commitments or diffusion outcomes, and combining innovation policy with competition policy in a coherent way.

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Appendix

A. Data Appendix

We utilized administrative data from Finland from 2014 to 2019, comprising financial statement statistics and business register information. The data coverage is extensive, and while there are some quality concerns in specific industries, it can be considered population data rather than a sample. To ensure relevance to the question, we restricted the data and reported all statistics about the filtered data. The filtering criteria are as follows: companies must have one or more employees, generate over 15,000 euros of revenue (as below this threshold, special tax treatment applies), be privately owned limited liability companies, and exclude the following sectors: Agriculture and Mining (A & B), Electricity, Gas, Steam and Air Conditioning Supply (D) and Water Supply; Sewerage, Waste Management and Remediation Activities (E), the Financial and Real Estate (K & L), Public Services (O), and Other Services (S-U). This subset encompasses a majority of Finnish firms that plausibly possess R&D and licensing potential. All monetary variables are deflated with CPI to 2015 euros and for building the indicators for extensive margins we use threshold values of 0.00001 for R&D expenditure over value added and 0.00001 for licensing income over gross output.

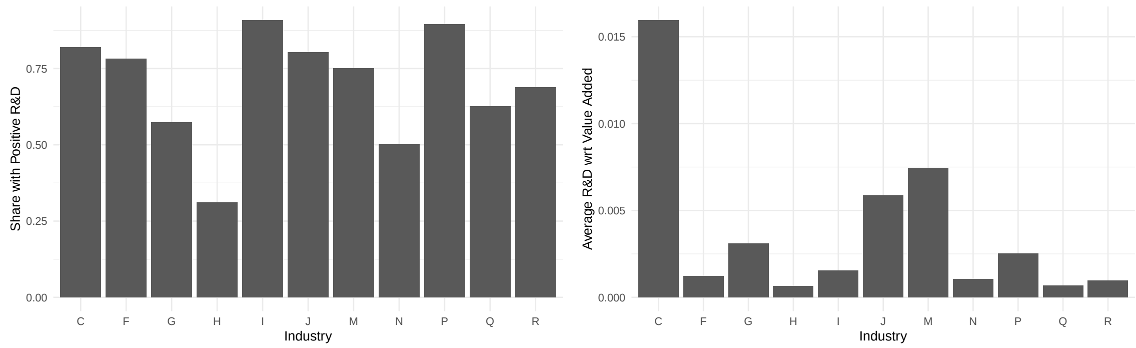

Figure 2: Average R&D with respect to value added (left) and probability of reporting positive R&D (right) on different industries

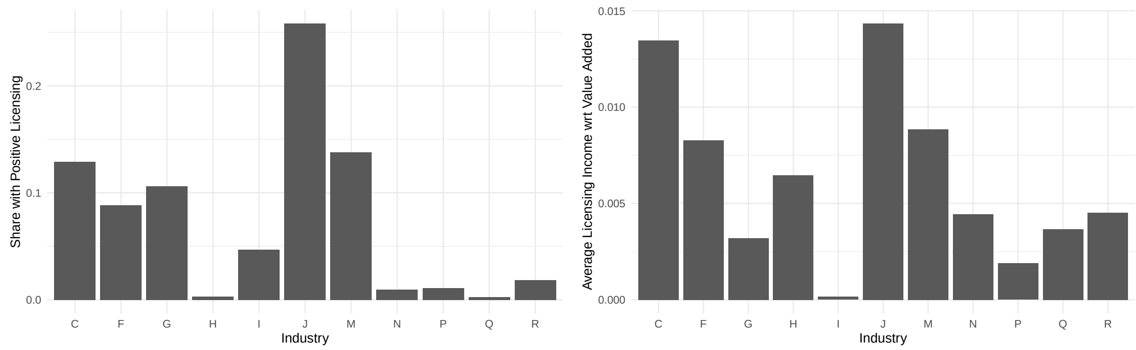

Figure 3: Average licensing income with respect to value added (left) and probability of reporting positive licensing income (right) on different industries

As the majority of the remaining industries do relatively modest amounts of R&D and have low level of licensing income, we narrow our focus. We base the filtering on information in Figures 2 and 3. In the figures we see that significant share of R&D and licensing connects to the industries Manufacturing (C), Information and Communication (J), Professional, Scientific and Technical Activities (M), Wholesale and retail trade; repair of motor vehicles and motorcycles (G), and Construction (F). Therefore, we limit our focus to the industries C, F, G, J, and M. We find these industries as most relevant for our study. However we also report robustness of the results with C, J, and M.

B. Empirical Appendix

B1: Additional Regressions

Table 8: Baseline vs IHS, in subset that has only R&D firms

Specification:	Baseline		IHS Specification	
Model:	(1)	(2)	(3)	(4)
Coefficients				
R&D expenditure	0.130	0.017	0.132	0.024
	(0.055)	(0.023)	(0.049)	(0.020)
(1-HHI)	-0.001	0.029	-0.002	0.025
	(0.002)	(0.011)	(0.002)	(0.009)
Details				
Additional controls	Yes	Yes	Yes	Yes
Year Fixed-effects	Yes	Yes	Yes	Yes
Firm Fixed-effects		Yes		Yes
Fit statistics				
Observations	225,076	225,076	225,076	225,076
R^2	0.003	0.354	0.008	0.470
Within R^2	0.003	0.001	0.007	0.002

Notes: Clustered standard errors at firm level in parentheses. Additional controls include total assets, equity and employment in logs.

Table 9: Baseline vs IHS, in subset that has only licensing firms

Specification:	Baseline		IHS Specification	
Model:	(1)	(2)	(3)	(4)
Coefficients				
R&D expenditure	0.514	0.051	0.331	0.092
	(0.108)	(0.040)	(0.088)	(0.051)
(1-HHI)	0.042	0.193	0.017	0.155
	(0.018)	(0.066)	(0.015)	(0.050)
Details				
Additional controls	Yes	Yes	Yes	Yes
Year Fixed-effects	Yes	Yes	Yes	Yes
Firm Fixed-effects		Yes		Yes
Fit statistics				
Observations	38,943	38,943	38,943	38,943
R^2	0.116	0.783	0.056	0.692
Within R^2	0.116	0.011	0.055	0.012

Notes: Clustered standard errors at firm level in parentheses. Additional controls include total assets, equity and employment in logs.

Table 10: Baseline vs IHS, in subset that has only industries C, J, and M

Specification:	Baseline		IHS	
Model:	(1)	(2)	(3)	(4)
Coefficients				
R&D expenditure	0.404	0.427	0.218	0.160
	(0.146)	(0.155)	(0.068)	(0.097)
(1-HHI)	0.012	0.042	0.001	0.031
	(0.009)	(0.013)	(0.005)	(0.011)
Details				
Additional controls	Yes	Yes	Yes	Yes
Year Fixed-effects	Yes	Yes	Yes	Yes
Firm Fixed-effects		Yes		Yes
Fit statistics				
Observations	140,808	140,808	140,808	140,808
R^2	0.101	0.630	0.041	0.485
Within R^2	0.101	0.113	0.040	0.014

Notes: Clustered standard errors at firm level in parentheses. Additional controls include total assets, equity and employment in logs.